社団法人　電子情報通信学会
THE INSTITUTE OF ELECTRONICS,
INFORMATION AND COMMUNICATION ENGINEERS

信学技報
TECHNICAL REPORT OF IEICE.

ルータレベルトポロジの構造特性とそのモデル化手法の提案

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あらまし　インターネットのトポロジ形状を計測した結果、ノードの出線数分布がパワー則（Power–Law）に従うことが近年明らかにされており、パワー則の性質を有するトポロジのモデル化手法の検討がなされている。しかし、モデル化手法により生成されるトポロジを経路制御などのネットワーク制御手法に適用するためには、出線数分布の一致のみならず、トポロジ構造の適切なモデル化が必要である。本稿では、ISP レベルのトポロジに着目したトポロジモデル化手法を提案する。まず、既存のモデル化手法で生成されるトポロジと ISP のトポロジの構造上の違いを明らかにする。その結果に基づいて物理的距離およびクラスタ係数に着目したトポロジ生成手法を提案し、その生成トポロジは経路制御手法の評価に適用可能であることを示す。

キーワード　Power-law、パワー則、トポロジ、ルータレベルトポロジ、経路制御、トポロジ生成モデル

Analyzing and Modeling Router–level Internet Topology and Application to Routing Control

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Abstract Recent studies on measurement studies on the Internet topology show that connectivities of nodes exhibit power–law attribute, but it is apparent that only the degree distribution does not determine the network structure, and especially true when we study the network–related control like routing control. In this paper, we first reveal structures of the router–level topologies using the working ISP networks, which clearly indicates ISP topologies are highly clustered; a node connects two or more nodes that also connected each other, while not in the existing modeling approaches. Based on this observation, we develop a new realistic modeling method for generating router–level topologies. In our method, when a new node joins the network, the node likely connects to the nearest nodes. In addition, we add the new links based on the node utilization in the topology, which corresponds to an enhancement of network equipments in ISP networks. With appropriate parameters, important metrics, such as the a cluster coefficient and the amount of traffic that pass through nodes, exhibit the similar value of the actual ISP topology while keeping the degree distribution of resulting topology to follow power–law.

Key words Power–law, Router–level Internet topology, ISP topology, AS topology, routing, modeling

1. Introduction

Recent measurement studies on Internet topology show that the connectivities of nodes exhibit a power–law attribute (e.g., see [1]). That is, the probability $p(k)$ that a node is connected to k other nodes follows $p(k) \sim k^{-\gamma}$. In recent years, considerable numbers of studies have investigated power–law networks whose degree distributions follow the power–law [2, 3] Here, the degree is defined as the number of out–going links at a node. The theoretical foundation for the power–law network is introduced in Ref. [4] where they also presents the Barabashi–Albert (BA) model in which the topology increases incrementally and links are placed based on the connectivities of topologies in order to form power–law networks.

However, even if the degree distributions of some topologies are the same, more detailed characteristics are often quite different. A pioneering work by Li et al. [5] has enumerated various topologies with the same degree distributions, and has shown the relation between the characteristics and performances of these topologies. With the technology constraints imposed by routers, the degree of nodes limits the capacity of links that are connected to. Li et al. point out that higher–degree nodes tend to be located at the edges of a network. Their modeling method in [5] provides a new insight in that the location of higher–degree nodes are not always located at the core of networks. Actually, different to AS–level topology, each ISP constructs its own router–level topology based on strategies such as minimizing of the mileage of links, redundancies, and traffic demands.

Although Li et al.'s approach is significant, it is insufficient for ISP networks. As will be discussed in Sec. 2., the Sprint topology and Abilene–based topologies are quite different in terms of the cluster coefficient. The main difference may come from the fact that scientific networks like Abilene provide fewer opportunities to enhance their network equipment because of budgetary constraints, while ISPs make their efforts on enhancement of networks based on their strategies. The difference can be also seen from the graphs of the Abilene network (Fig. 6 (e) of Ref. [5]) and the Sprint network (Figs. 7 and 8 of Ref. [6]). More importantly, these differences greatly affect methods of network control. One typical example is routing control. In our previous work [7], we have examined how topology's structural differences affects throughput performance using minimum hop routing and optimal routing methods. The results show that in the ISP topology we examined, the optimal routing method gives smaller the maximum link utilization (about $1/3$) compared with minimum hop routing, while a topology by the BA model achieves much smaller of the maximum link utilization (about $1/10$). These results indicate that the link utilization in the router–level topology is much far from the one in the conventional modeling method. The same argument could also be applied to the higher–layer protocols. That is, for vital network researches, a modeling method for a realistic router–level topology is needs to be developed.

In this paper, we develop a modeling method to construct ISP router–level topologies. To achieve this, we first reveal basic structures for the router–level topologies other than the power–law property of degree distribution. The results clearly reveal the ISP topologies had a much higher cluster coefficient than the AS topology [8], the topology examined by Li et al. [5], and the other topologies attained with conventional modeling methods. We therefore propose a modeling method for realistic router–level topologies. Our modeling method has two main features. When a new node joins the network, the ISP likely connects it to the nearest nodes, while the ISP add new links based on the utilization of nodes. With our modeling, important topology–related metrics such as the amount of traffic passing through nodes have almost the same characteristics as the actual ISP topologies with appropriate parameter settings, while still keeping the degree distribution of the topology to follow the power–law. We also apply our routing method in Ref. [7] to the topology generated by our modeling method, in order to demonstrate that our modeling method constructs the realistic router–level topology, and can be actually used for evaluations on routing control. The results show that the characteristic of link utilization is similar to the actual ISP topology.

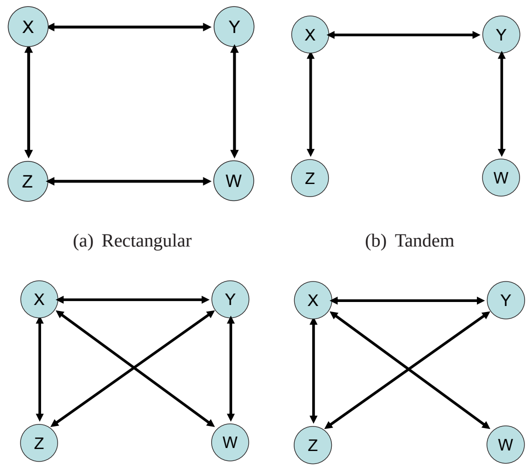
This paper is organized as follows. Section 2. discusses the basic structure of ISP's router–level topologies. We then discuss our development of a new modeling method in Section 3. to obtain realistic router–level topologies that can be applied to "traffic flow" level research. Sec. 4. concludes the paper.

2. Basic Properties of Router–level Topology

In this section, we investigate the structure of router–level topologies as a first step to modeling a router–level topology, and discuss the differences between actual ISP's router–level topologies and topologies generated by existing modeling methods.

2.1 Network Motif

Milo et al. [9] have introduced the concept of *Network Motif*. The basic idea is to find several simple structures in complex networks. In this paper, we select four–node subgraphs as building blocks for router–level topologies following the Milo et al.'s approach, i.e., rectangular (Fig. 1(a)), tandem (Fig. 1(b)), sector (Fig. 1(c)), umbrella (Fig. 1(d)), and full–mesh. The case of a three–node subgraph, which has an exactly the same meaning as "cluster", will be discussed later. Figure 2 plots the frequency of four–node subgraphs appearing in each topology. The labels along the horizontal axis represent the ISP networks (from ISP1 to ISP7) that have been measured with Rocketfuel tools [6]. A topology generated by the BA model (Model1), such that the number of nodes and links is the same as that for the Sprint topology is also presented. The results from the Ailene–based topology used in Ref. [5] (Model2) is also plotted in the figure. We also show the results obtained by a AS–level topology from INET topology generator (Model5 in Fig. 2), and topologies generated by conventional modeling methods (Model3 by the BA model, and Model4 by the ER model [10] in which links are randomly placed between nodes) for comparison. Models 3, 4, 5 have the same number of nodes and links. We can see that: 1)



(a) Rectangular (b) Tandem

(c) Sector (d) Umbrella

Figure 1 Four–node subgraphs

there are many more "sectors" with the Sprint topology (ISP1) than with the BA topology (Model1), 2) "full-mesh" appears more often than model topologies in the router–level topologies of ISPs (Sprint, abovenet, AT&T, ebone exodus, level3, verrio), 3) the percentile sum for "rectangle", "umbrella", and "sector" is large (around 30%) for ISP topologies while not for model topologies.

From the figure, it is quite apparent that router–level topology is very different to the AS–level topology and the topologies generated with conventional modeling methods. Furthermore, ISP–level topologies (from ISP1 to ISP7) are highly clustered compared with the Abilene–based topology (Model2) presented by Li et al. [5]. We conjecture that the reason for differences derives from redundancy considerations in building the ISP networks. In what follows, we concentrate on the Sprint Topology (ISP1) and investigate the router–level topology in detail.

2.2 Detailed analysis of router–level topology

To compare how the previously–discussed structure for router–level topology affects the basic properties of networks, we prepare three topologies that have the same number of nodes and links. For the router–level topology, we use ISP1 (Sprint). Two topologies generated by the BA model (Model1 in Fig. 2) and the ER topology generated by the ER model are also used for purposes of comparison. The degree distributions for these three topologies were presented in Ref. [7], where we can confirm that the degree distribution for the Sprint topology follows a power-law.

We use the following metrics for node i to investigate the characteristics of topologies:

$A(i)$, $D(i)$: Average and maximum number of hop–counts from node i to all other nodes. Hereafter, we will call the maximum hop–counts as diameters.

$C_e(i)$: Cluster coefficient [11] for a node, which is defined as

$$C_e(i) = \frac{2E_i}{d_i(d_i-1)},$$

where d_i is the degree of node i, and E_i is the number of links connected between node i's neighbor nodes.

We also consider two centrality measures; degree centrality and betweenness centrality [12]. For each node i, degree centrality is defined as the degree of node i, and betweenness centrality is defined as the number of node–pairs that pass through node i.

The cluster coefficient for each node is ranked in ascending order in Fig. 3(a). In the figure, the results of the Abilene topology are

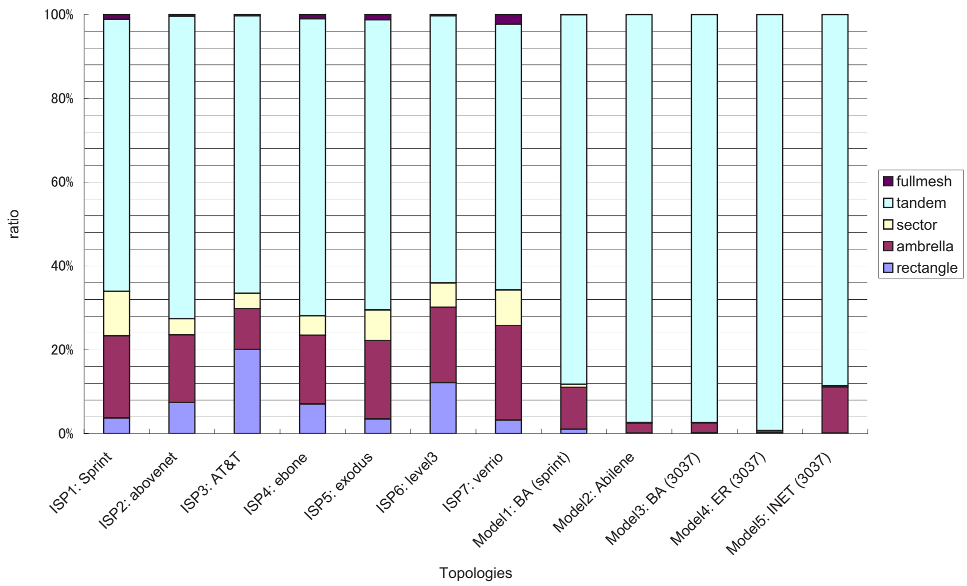
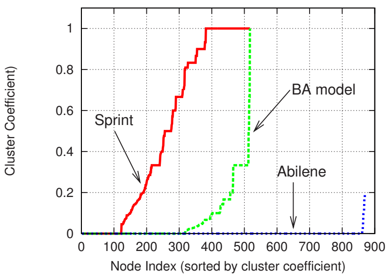
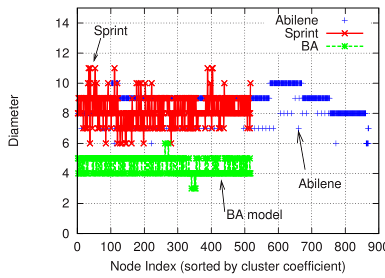


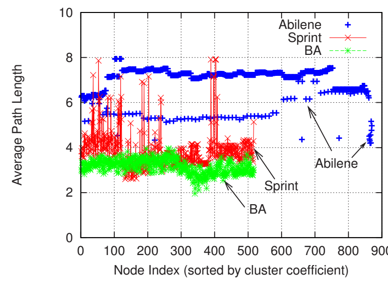
Figure 2 Distribution of four–node subgraphs



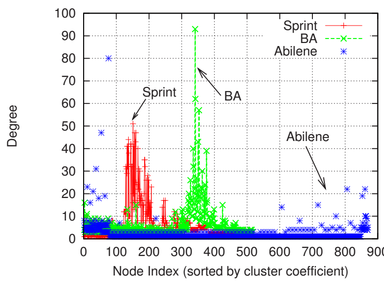
(a) Cluster coefficient $C(i)$



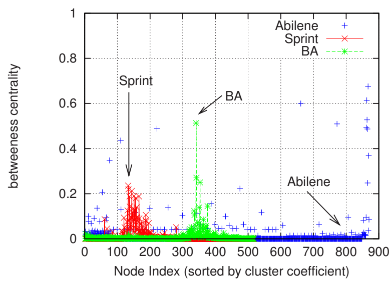
(b) Diameter $D(i)$



(c) Average path length $A(i)$



(d) Degree centrality



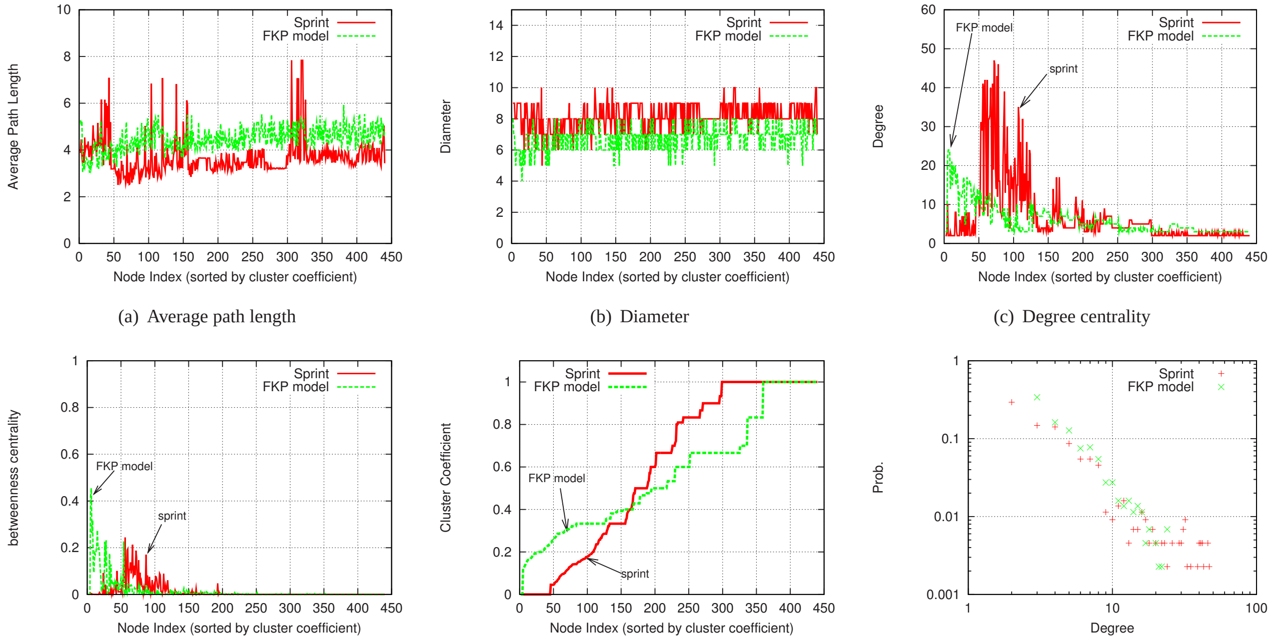
(e) Betweenness centrality

Figure 3 The basic properties of the router–level topology: Comparison among the Sprint, BA, and Abilene topologies

also presented. We can see that the cluster coefficient for the Sprint topology is much larger than that for the BA topology. Furthermore, the results in Figs. 3(a) and 3(d) show that lower–degree nodes are more highly clustered with the Sprint topology; a node with two out–going links always forms a cluster, while higher–degree nodes do not always have a high cluster coefficient. Other interesting observations can be seen in Figs. 3(b) and 3(c), which show the diameter $D(i)$ and average distance $A(i)$ from each node; both with the Sprint topology are larger than those with the BA topology. A node in the BA model tends to be connected to higher–degree nodes, and therefore any two nodes communicate with smaller hop–counts via the higher–degree nodes. However, the results for the router–level topology do not exhibit this effect. Since the average distance with the Sprint topology is larger than that with the BA topology, the small world property no longer hold with the router–level topology. Therefore, another attachment metric, rather than the degree–based metric, has to be considered to model the router–level topology, which we will discuss and propose in Section 3.. The Abilene topology shows quite different characteristics in Fig. 3(a). With the Abilene topology, the cluster coefficient is even lower than the BA topology, and the average path length is much longer than the Sprint topology and the BA topology. The reason for this is apparent in that the Abilene topology is three–level hierarchical topology.

3. Modeling Methodology for Router–level Topologies

The results in the previous section revealed that ISP–level topologies are very different to topologies using conventional modeling methods in that: 1) the cluster coefficient for lower–degree nodes



(a) Average path length (b) Diameter (c) Degree centrality

(d) Betweenness centrality (e) Cluster Coefficient (f) Degree distribution

Figure 4 FKP model: $\alpha = 40$ (used in [13]).

is high, and 2) improved maximum traffic demand between nodes achieved by the optimal routing with the ISP topology is less than that in the BA topology [7]. This clearly indicates that ISP topologies are *locally* clustered networks, i.e., each node is connected to geographically closer nodes, and thus topologies attained by conventional models that do not use geographic information cannot appropriately evaluate for network control mechanisms, such as routing control.

Fabrikant et al.'s FKP model in Ref. [13] is a method that incorporates geographical information. However, they did not discuss in Ref. [13] whether the topologies resulting from the FKP model matches Internet topologies or not. The original FKP model, which adds one link for each node arrival, actually has numerous one–degree nodes [14], and is very different to the AS topology as shown in [15]. A question naturally arises as to whether the FKP model can actually predicts router–level topologies or not. In this section, we show that although topologies obtained with the FKP model are close to router–level topologies, they still have a lower cluster coefficient and do not match betweenness centrality. We therefore propose a new modeling method to generate router–level topologies in Sec. 3.2.

3.1 FKP topology: distance–based modeling

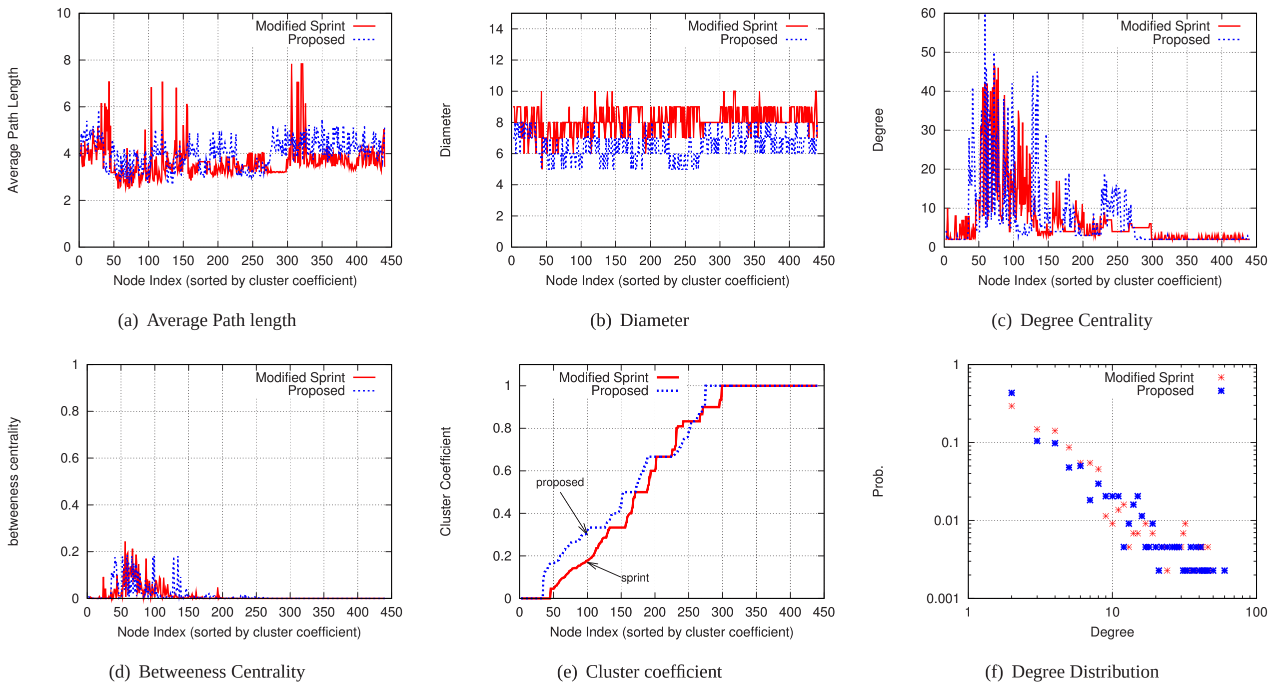
The FKP model proposed by Fabrikant et al. [13] revealed that the power–law property of degree distribution can still be obtained by minimizing "distance" metrics. This model does not use preferential attachment to add links, and instead uses minimization–based link attachment. More specifically, the FKP model works as follows. Each new node arrives at randomly in the Euclidean space $\{0, 1\}^2$. After arriving at new node i, the FKP model calculates the following equation for each node, j, already existing in the network: $\alpha \cdot w_{ij} + l_{0j}$, where w_{ij} is the Euclidean distance (i.e., physical distance) between nodes i and j, and l_{0j} is the hop–counts distance between node j and a pre–specified "root" node (node 0). α is a parameter that weights the importance of physical distance. If α has a lower value, each node tries to connect to higher degree nodes; $\alpha = 0$ is an extreme scenario that creates a star–topology. If α has a higher value, each node tries to connect their nearest nodes. A topology with high a α is shown to behaves like an ER topology. The power–law property of the degree distribution appears at a moderate value of α value. Here, there are several hub–nodes in each region, and the hub–nodes form a power–law.

Figure 4 compares the ISP topology with the FKP model with regard to the same properties we previously discussed. In the figure, we do not use the actual Sprint topology (ISP1), but we modified the Sprint topology by eliminating one–degree nodes and their corresponding link since one–degree node has no impact on routing control. The resulting topology has 439 nodes / 1516 links, and the average degree is 3.46. In obtaining the results of the FKP topology, we add three links when each node arrived in order for setting the total number of links so that it is almost the same as for the modified Sprint topology. For the initial graph G_{init}, we use the 14–node NSFnet topology with geographic latitudinal and longitudinal information. The value for α is set to 40 as used in Ref. [13].

A first impression of the results for the FKP topology is that the shape is closer than the results for the BA topology (see Figs. 3(a) through 3(e)). However, a clear difference appears again in the cluster coefficient; although the FKP model constructs a more highly–clustered network than the BA topology, the cluster coefficient is still smaller in lower–degree nodes. Another difference is that the maximum degree of the FKP topology is low. Note that the maximum degree depends on the parameter setting. As α gets smaller, the maximum degree can be increased. However, at the same time, a smaller value of α leads to a star–like topology and the betweenness centrality also becomes larger than the value in Fig. 3(e). Therefore, in the FKP model, fitting the degree distribution by appropriate α results in mismatches on the betweenness centrality of the modified Sprint topology.

3.2 New modeling method for router–level topologies

The fact that the FKP model cannot construct router–level topologies because of much larger betweenness centrality drives us to develop a new modeling method by extending the FKP model. Our model incorporate the physical distance between nodes following the FKP model. However, unlike the FKP model, we also incorporate the enhancement of network equipments in ISP networks. For this, we add new links based on node utilization in the topology. However, the problem is where to place the new link. In this paper, we select a node that have the largest betweenness centrality in the network, and then attach a link between neighboring nodes. From



(a) Average Path length (b) Diameter (c) Degree Centrality
(d) Betweeness Centrality (e) Cluster coefficient (f) Degree Distribution

Figure 5 Results with proposed modeling method: α is 25, and β is 200.

the view point of graph theory, adding links to neighboring nodes increases to increase the cluster coefficient of the topology. From the view point of network design, on the other hand, this corresponds to improve reliability against network failures (e.g., link failures). It also corresponds to decreasing utilization of nodes in the topologies; some part of the traffic that has passed through the most utilized node is rerouted via added links.

More specifically, our algorithm works as follows. For a given initial network $G_{init}(V, E)$, when a new node joins the network, m links from that node are added (network growth). Besides, k links with no relation to m links are added based on node utilization of the network, which corresponds to network enhancements by ISPs (network enhancement). This procedure is continued until n nodes are added to the initial network. Since m links and k links are added to the network at each of node join, the resulting topology has $\|E\| + n \cdot m + k$ links, where $\|E\|$ is the number of links in the initial network. In the following, we explain the link attachment policy for network growth (m–link addition) and policy for network enhancement (k–link addition).

3.2.1 Network growth model

Step 0: Set the initial network.

Step 1: For each node i ($\in V$) already existing in the network, calculate the attachment cost to node i as

$$\alpha \cdot w_{ij} + \bar{h}_i, \qquad (1)$$

where $\bar{h}_i$ is the average distance from node i to the other nodes.

Step 2: Select m nodes in an ascending order by Eq. (1). Then add one link to each of selected nodes.

Step 3: Go back to Step 1, until the number of nodes reaches n.

3.2.2 Network enhancement model

Add k links via the following steps.

Step 1: Calculate betweenness centrality for each node in the network, and then select a node, x, that has the largest betweenness centrality in the network.

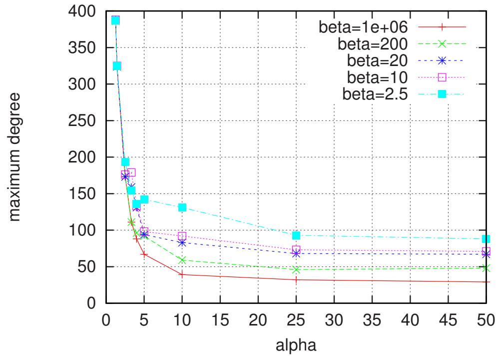


Figure 6 Effect of α and β

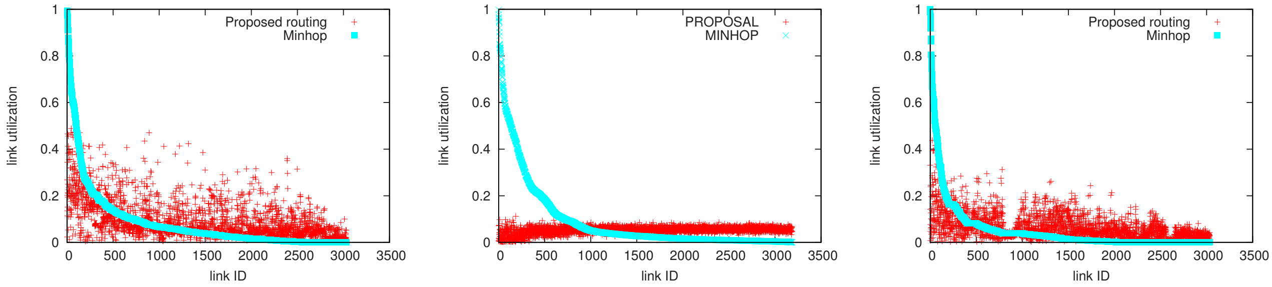
Step 2: From the set of neighbor nodes from x, select two nodes y and z, that minimize,

$$\begin{cases} \beta \cdot w_{yz} + (1/D_z), & \text{if } D_z > D_y, \\ \beta \cdot w_{yz} + (1/D_y), & \text{otherwise,} \end{cases} \qquad (2)$$

where β is the parameter for weighting importance to the physical distance, and D_p denotes the betweenness centrality of node p. Note that by using the equation $1/D_p$, more traffic on node x is rerouted via the link between node y and z.

3.3 Evaluation on Modeling method

We show the results with our modeling method in Fig. 5. Here, the number of joining nodes n is set to 425, and we use $m = 2$, i.e., when each node arrive, two links are prepared for newly arriving node. We set $k = 649$ so that the resulting topology has the same number of nodes (439) and links (1519) as the modified Sprint topology. If a one–degree node is necessary, the original FKP model that connects one link for node arrival can be applied. For the initial graph G_{init}, we use the NSFnet topology with geographic latitudinal and longitudinal information. By setting parameters α and β to be 25 and 200, the resulting topology is very close to the Sprint topology for both degree distribution and betweenness centrality.



(a) Sprint topology (b) BA topology (c) the topology obtained by our modeling method

Figure 7 Distribution of link utilizations by applying the routing method in Ref. [7]

Note that we show the best parameter settings for the topology that looks like the modified Sprint topology in Fig. 5. Actually, depending on α and β, the topology differs from Fig. 5. To see the impact of parameter settings, we show the maximum degree dependent on α for each β in Fig. 6. Apparently, inherited parameter α from the FKP model shows the same tendency as presented in Ref. [13]; as α get smaller, the topology becomes a star–like topology. That is, if the maximum degree equals to n $(= 425)$, the topology becomes the star topology. β also impacts on the maximum degree in the topology; the maximum degree become larger as β gets smaller (i.e., weights on the physical distance becomes smaller). Considering that the maximum degree in the modified Sprint topology is 47, α should be greater than 20 and the β greater than 200, to generate a realistic ISP topology with a moderate maximum degree.

We finally show the link utilization of the topology generated by our modeling method. In Fig. 7, we show the distribution of link utilization for the modified Sprint topology (Fig. 7(a)), BA topology (Fig. 7(b)), and the topology obtained by our modeling method (Fig. 7(c)). The vertical axis shows link utilization, and the horizontal axis represents link index. The link index is given in an ascending order of link utilization when the minimum hop routing method is used. Then, the link utilization of the routing method in Ref. [7] is shown for each link index. Note that, in obtaining these figures, we assume that traffic demand between nodes is identical for every node–pair, and use the link capacity assignment algorithm in Ref. [7]. From Fig. 7(a) and Fig. 7(c), we observe that the distribution of link utilization in our topology is quite similar to that in the modified Sprint topology for both minimum hop routing and routing method in Ref. [7], while the link utilization of the BA topology is quite different from that of the modified Sprint topology.

4. Concluding Remarks

For vital network researches, a method for modeling the realistic router–level topology urgently needs to be developed. However, we have shown that the structure of ISP topologies is quite different from that of topologies achieved with conventional modeling methods. Based on this, we have developed a new realistic modeling method for generation of router–level topologies. In our method, when a new node joins the network, it likely connects to the nearest nodes. In addition, we added new links based on node utilization in the topology, which corresponded to enhancing network equipments in ISP networks. The evaluation results have shown that our modeling method achieve a good compatibility with the Sprint topology with regards to degree distribution and the amount of traffic passing through nodes.

In this paper, we have concentrated on the routing control as one of network control mechanisms, and have proposed a modeling method for router–level topology that can be applied to the routing control. However, for the higher–layer protocols, it may require more detailed modeling. Actually, the link capacity model used in our work is not the optimal one, which may give much impact on studies of higher–layer protocols such as flow control. One of our future works is to reveal correlation between capacity and degree, and then consider the appropriate models for link capacity assignments in router–level topologies.

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