PAPER Special Section on New Generation Network towards Innovative Future Society. Self organizing topology transformation for Peer-to-Peer (P2P) networks

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SUMMARY

Topological structure of peer-to-peer (P2P) networks affects their operating performance. Thus, various models have been proposed to construct an efficient topology for the P2P networks. However, due to the simultaneous failures of peers and other disastrous events, it is difficult to maintain the originally designed topological structure that provides the network with some performance benefits. For this reason, in this paper we propose a simple local rewiring method that changes the network topology to have small diameter as well as highly clustered structure. Moreover, the presented evaluation study shows how these topological properties are involved with the performance of P2P networks. *key words:* Self-Organization, topology transformation, rewiring, Peer-to-Peer networks.

1. Introduction

Four decades ago the Internet was just a small-scale research network with a few hundred users only. Now it becomes large enough to accommodate more than a billion of users and various types of network applications. Especially, the growth of users' demand on new network services forces the Internet to embrace new network design paradigms, such as Peer-to-peer (P2P) networks. P2P networks are operated in a distributed manner that is different from the conventional centralized networks. This characteristic of P2P networks enables users to diverse connectivity and share network resource more efficiently among participants. For the reasons, P2P networks have gained much interest among network researchers in recent years.

P2P networks can be categorized into two groups in terms of its topology construction method; one is called structured, and the other is unstructured P2P networks.

The former constructs its structure in a strictly controlled way using Distributed Hashing Tables (DHTs) that enables a peer to find a file easily without flooding queries through a network. Chord [1], Pastry [2], and CAN [3] belong to this category. Although, it can achieve the best performance in terms of searching a desired information from this structured network, this scheme suffers from frequent joins and departures of peers that cause heavy control overhead and deteriorate system performance [4] so that it has been adapted in the Internet with some limitation [5].

On the other hand, the latter approach employed by Gnutella [6], and KaZaa [7] builds its structure by connecting peers randomly. Surprisingly, this random approach has been widely accepted in the current Internet because its random interconnection among peers enables the network to react quickly against the scenarios of peer's failure or attacks. However, the random topology does not guarantee anything other than an efficient topology [4].

Topological properties of P2P networks affect their performances. For instance, degree distribution of the network (Refer to Section. 2.1) is involved with its robustness [8], clustering coefficient (Refer to Section. 2.2) is related to the spread of malicious code [9], and network diameter (Refer to Section. 2.3) is highly related to the searching efficiency in P2P networks [10]. Thus, P2P networks can take advantage of their topological structures if they can be maintained properly.

There are some algorithms to construct a network structure with desired topological properties in physics community, such as Erdos & Renyi (ER) random model [11], Watts & Strogatz (WS) small world model [12], and Barabási & Albert (BA) scale free model [13]. Especially, the BA model is known to construct a scale free or power law topology (Refer to Section. 2.1) that provides high search efficiency. Due to the reason, some efforts [4][14] have been devoted to take advantages of this power law structure in the construction of a P2P network topology.

Although the topology construction methods can construct a small diameter topology of which P2P networks can take advantage to achieve high search efficiency, some extreme cases such as disastrous multiple failures on peers may distort its structure severely so that inherent topological advantages may be removed. In such a case, some mechanisms are required to restore the distorted topological structure.

For this reason, in this paper we propose a rewiring method to transform the topological structure of P2P network (e.g., possibly distorted by some disastrous multiple failures on peers) in a self organizing manner so that the P2P network obtains certain topologi-

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cal properties (small diameter or high clustering coefficient(Refer to Section 2.2)). P2P networks can benefit from both topological properties. For instance, a topology with high clustering coefficient is known to handle heavy traffic more efficiently [18], and high searching efficiency can be achieved on a small diameter topology. Importantly, the proposed method is implemented in a self organizing manner which means that topological structure of a network can be transformed simply by interaction among neighbor peers without help of centrally dedicated control unit. The performance of rewired topologies are evaluated in terms of searching efficiency that is highly required for P2P networks. We also demonstrate how a truncated power law topology (Refer to Section. 2.1) can be constructed from the proposed rewiring method. The truncated power law topology solves one known problem in a general power law topology that a few hub nodes suffer from high traffic load which is not desirable phenomenon for P2P networks.

The rest of this paper is organized as follows. In Section 2, we describe some basic properties that characterize a topology, namely degree distribution, clustering coefficient, and network diameter. Section 3 presents the detail description of the proposed rewiring algorithm. This is followed by a theoretical analysis on the degree distributions of topologies that are rewired from the proposed method in Section 4. In Section 5, numerical results are provided for the evaluation of the proposed algorithm. Finally, we conclude the paper in Section 6.

2. Basic topological properties

Anything represented as a structure consisting of nodes and links can be analyzed using the theory of complex networks. Since the proposed algorithm transforms a network structure, the complex network theory is a useful tool to investigate the performance of the proposed rewiring method. In this section we summarize some quantities and measures of complex networks that we use for characterizing the transformed topologies.

2.1 Degree distribution

A single node of a network can be characterized by its degree. The degree k_i of a node i is defined as the total number of links that are started from the node i. The spread of degrees of all nodes in a network is characterized as a distribution function P(k) that is the probability that a randomly chosen node has degree k. When degree distribution of a network follows a power function shown in equation (1), the network is called a power law or scale free network.

$$P(k) \sim k^{-\gamma} \tag{1}$$

A power law topology is known to have two interesting properties, namely small diameter and robustness, which are desirable topological properties for P2P networks. Thus, some methods such as "Phenix" [4] and "LLR" [14] have adopted the topological structure for the construction of a P2P network topology.

On the other hands, there is a criticism on the use of a power law topology for P2P networks since a few hub peers in the power law topology suffers from large number of degrees that cause over-loaded traffic on those peers. Thus, it is desirable to limit the maximum number of links that each peer has. We call a power law topology whose peers have the limited maximum number of links as "a truncated power law topology" in this paper.

2.2 Clustering coefficient

This property quantifies how well neighbor nodes of a given node are connected each other. The clustering coefficient C_i of a node i is defined as the fraction ratio between the existing links and possible number of total links among the neighbor nodes of the node i. For instance, when a node i has k_i neighbor nodes and there are E_i number of links among k_i nodes, the clustering coefficient C_i of node i is defined as follows,

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \tag{2}$$

Thus, the average clustering coefficient (ACC) of a topology is simply calculated by averaging the clustering coefficients of all nodes in the topology. Under the assumption that two topologies have the same number of nodes and links, we can infer that a topology with high average clustering coefficient tends to have strong modularized or clustered structure because some parts of the topology are well connected while the other parts are loosely connected. Thus, it implies that a topology with high average clustering coefficient can take advantages of clustered structure such as delaying virus spreading.

2.3 Network distance

Network distance d_{ij} represents the number of links between two nodes *i* and *j* along the shortest path connecting them. This topological property has been used to measure the efficiency of methods for a P2P overlay network construction because of the relation between this property and query transmission time among peers. There are two popular ways to construct a topology with a small distance. One is based on the rewiring process introduced by Watts et al [12] (It is called a small world model). The other method is to construct a power law topology. The difference between two approaches is that the former makes use of a rewiring mechanism and builds a homogeneous network (all nodes have approximately similar number of links) while the latter is based on a network growing mechanism and builds a heterogenous network. The average shortest path (ASP) of a topology is simply calculated by averaging the network distances of all pairs in the topology.

3. Proposed rewiring algorithm

Fig. 1 illustrates the proposed rewiring process. In the figure, a randomly chosen peer in the network requests a neighbor peer to pass the identification (ID) of one of its neighbors.

A random peer can be chosen in various ways. Firstly, a dedicated hardware that maintains the identifications of existing peers in the networks can be used to select a random peer from the network. Although this approach provides the best performance, it may suffer from a single point failure or may not agree with one of the contributions in this paper which is the use of self organizing mechanism. The second choice can be to use some algorithms such as a random walk approach proposed by Vishnumurthy et al [15]. In this paper we do not consider the issue of random peer selection further since we assume that a random peer can be chosen by either one of the above methods.



Fig. 1 Illustration of the proposed rewiring method. Either (a) or (b) is implemented each time with a probability of β and $(1-\beta)$, respectively.

In Fig. 1(a), let us assume that a randomly chosen peer A requests a neighbor peer E to pass the ID of one of its neighbors (F, G). Let's assume that the peer Epasses the ID of peer G as a response, then the peer Adisconnects the link to peer E and rewires to peer G. On the other hands, in Fig. 1(b) a randomly chosen peer A passes the ID of one of its neighbor peer to another randomly chosen peer. In this example, the peer A passes the ID of peer B to a randomly chosen neighbor E. Then, peer E disconnects the link to the peer A and rewires to peer B.

The design intention to introduce two rewiring schemes shown in Fig.1(a) and Fig.1(b) is that the peer A and G in Fig. 1(a), and peer B and E in Fig. 1(b) have different degree characteristic. Before we explain the reason, we need to understand one intuitive idea that "a neighbor peer of a randomly chosen peer tends to have large degrees". It is because a high degree peer has many neighbors so that a randomly chosen peer tends to be one of high degree peers' neighbors.

Based on the understanding of the idea, in the first scheme, the neighbor peer E of the randomly chosen peer A loses a link, and the link is connected to the peer G. In other words, the high degree peer E (a neighbor of a randomly chosen peer) loses a link and the link moves to a randomly chosen peer G whose degree state is unknown. Thus, we can consider this process as a link taken from a high degree peer is added to a randomly chosen peer. The second scheme is opposite to the first scheme. The randomly chosen peer A loses a link and the link is added to the high degree peer B (a neighbor of a randomly chosen peer).

Thus, by combing the both schemes we are able to manipulate the degree distribution of the network. For this reason, we make use of a probability β to select one of both schemes. The first scheme is chosen with the probability β , and the second scheme is chosen with the probability $(1 - \beta)$.

Lastly, we introduce two more parameters for this rewiring method, namely M and n that represent the maximum and minimum number of links that a peer is allowed to have. Thus, a peer refuses the request of connection or disconnection when its degree is bounded to M or n, respectively.

4. Theoretical analysis



Fig. 2 State transition rate diagram that shows the variation of the number of nodes with a certain degree.

We analyze the degree distribution of the topology that is transformed using the proposed rewiring method with three parameters, β , M, and n.

Let us define N_k as the number of peers that has degree k. Since a randomly chosen peer is rewired every time unit, N_k keeps changing as the time passes. For instance in Fig. 1(a), when the peer E loses a link, N_3 is reduced by one, and at the same time N_2 is increased by one. In addition, when the peer G gains the link, N_2 is increased by one, and at the same time N_1 is reduced by one.

We describe the variation of N_k in the network as

the state transition diagram shown in Fig. 2. The state diagram begins with N_n and ends with N_M since n and M are the minimum and maximum number of degrees that a peer is allowed to have in our rewiring model. Two rate coefficients are involved in this process which are λ_k and μ_k , the former and the latter represent rates of the attachment and the detachment of a link to a peer with degree k, respectively. In the equilibrium state, the influence tending to move from i^{th} state to j^{th} state must be equal to the influence tending to move from j^{th} state to i^{th} state. Thus, by equating the influences that cross the dot lines between adjacent states in Fig 2, we obtain equation (3) as follows,

$$\lambda_n N_n = \mu_{n+1} N_{n+1}$$

$$\vdots$$

$$\lambda_{k-1} N_{k-1} = \mu_k N_k$$

$$\lambda_k N_k = \mu_{k+1} N_{k+1}$$

$$\vdots$$

$$\lambda_M N_{M-1} = \mu_M N_M$$
(3)

Since the probability $P(k)^{\dagger}$ is defined as N_k/N where N is the total number of nodes, multiplying 1/N in both sides of the equation (3), we finally obtain equation (4).

$$P(k) = \frac{\lambda_{k-1}}{\mu_k} P(k-1)$$

= $\frac{\lambda_{k-1}\lambda_{k-2}\cdots\lambda_n}{\mu_k\mu_{k-1}\cdots\mu_{n+1}} P(n) \quad (n \le k \le M) \quad (4)$

To simplify the equation (4), we define the coefficients of attachment λ_k and detachment μ_k as follows,

$$\lambda_k = ak\beta + (1 - \beta)k/(\langle k \rangle N) \tag{5}$$

$$\mu_k = bk\beta + (1-\beta)/(N-N_n) \tag{6}$$

where a and b are constants, and < k > is the average degree.

In Fig.1(a), the node E loses a link and the node G gains the link while the randomly chosen node A maintains its degree. Since both nodes E and G are selected by one of its neighbor nodes (node E is a neighbor of the randomly chosen node A, and node G is a neighbor of the node E), if they have large number of neighbors (degrees), they have high probability to be chosen to lose or gain a link. Thus, we assume that the attachment coefficient λ_k and the detachment coefficient μ_k are proportional to its degree k. Thus, this relation is defined as $ak\beta$ and $bk\beta$ in the first terms on the right side of the equations (5) and (6).

In Fig.1(b), the randomly chosen node A loses a link, and its neighbor node B gains the link. Thus, the detachment coefficient μ_k (probability that a randomly chosen node loses a link) becomes $1/(N - N_n)$ (nodes

with degree n do not lose a link). On the other hands, the attachment coefficient λ_k (probability that a node at the end of the randomly chosen link is chosen to gain a link) becomes k/L where L is the total number of links (L=<k>N). Thus, they are shown as $(1-\beta)k/(<k>N)$ and $(1-\beta)/(N-N_n)$ in the second terms on the right side of the both equations (5) and (6).

Substituting the equations (5) and (6) into the equation (4) gives

$$P(k) = \frac{(a\beta < k > N + (1 - \beta))^{k - n}}{\prod_{i=n+1}^{k} (ib\beta(1 - P(n))N + (1 - \beta))} (\frac{1 - P(n)}{< k >})^{k - n} (k - n)! P(n)$$
(7)

• underlineCase 1. $(\beta=1.0)$ By substituting $\beta=1.0$ into the equation (7), P(k) is simplified as follows,

$$P(k) = \frac{n}{k} \left(\frac{a}{b}\right)^{k-n} P(n) \tag{8}$$

In the equation (8), there are two unknown variables, namely, the ratio of a/b, and P(n). To find the variables, the following relations are formulated.

$$\sum_{k=n}^{M} P(k) = 1 \tag{9}$$

$$\sum_{k=n}^{M} kP(k) = \langle k \rangle \tag{10}$$

Finally, substituting the equation (8) into the equations (9) and (10), the two unknown variables can be calculated.

 $\frac{\text{Case 2 } (\beta=0.0)}{\text{By substituting } \beta=0.0 \text{ into the equation (7), } P(k)$ is simplified as follows,

$$P(k) = \left(\frac{1 - P(n)}{\langle k \rangle}\right)^{k - n} (k - n)! P(n)$$
(11)

To find the initial value of P(n), we substitute the equation (11) to the equation (10).

• Case 3 $(0.0 < \beta < 1.0)$ When β is not equal to either 0.0 or 1.0, it is complicated to simplify the equation (7). Due to the similarity between the equation (11) and the equation (7), we just conjecture that the degree distributions is similar to the case 2.

4.1 Numerical confirmation

Fig.3 confirms the accuracy of the previous analysis. We initially generated random topologies whose degree distributions follow the poisson distribution and rewired them with different values of β (case 1 and 2). The degree distributions of the rewired topologies are

[†]Equation (1) - the probability that a randomly chosen node has degree k.



Fig. 3 The degree distributions from analytical (dot lines) and numerical (circles) results. For the simulation, an Erdos and Renyi (ER) random topology [11] with the total nodes N of 10^4 and the average degree $\langle k \rangle$ of 10 is rewired by limiting the maximum and the minimum number of degrees M and n to 200 and 1, respectively.

shown as circles, and analytically obtained degree distributions from the equations (8) and (11) are shown as dot lines. Both numerical and analytical results are well matched. For the case 3, we observed that the degree distribution is similar to the case 2. However, in spite of the similarity in their degree distribution, we observe that they have different value of the average shortest path (ASP) or the average clustering coefficient (ACC). We present the result in the next section.

5. Simulation Experiments

In this section, we show how basic topological properties as well as searching efficiency vary depending on the parameter β of the model. Then, the convergence of the topological transformation is investigated. Lastly, we demonstrate how to transform a topology into a truncated power law topology from the proposed rewiring algorithm.

5.1 Analysis of basic topological properties

Fig. 4 plots the variation of two topological properties (ACC and ASP) as a function of the number of rewiring times. The number of rewiring times starts from 10^4 . It is because the topology we used for this simulation has 10^4 nodes so that carrying out one rewiring per node causes rewiring 10^4 times totally. The value β represents the combination ratio between two cases shown in Fig. 1(a) and Fig. 1(b). We call the former and the latter as the first case, and the second case from now on.

5.1.1 Average clustering coefficient(ACC)

• A topology that grows with a joining



Fig. 4 Variation of topological properties of the rewired topologies with different values of β as the number of rewiring times increases. ACC: Average Clustering Coefficient, ASP: Average Shortest Path. Erdos and Renyi (ER) random topology [11] is used as an original topology, and then it is rewired $(M=\infty, n=1)$.

method tends to have extremely small ACC as the number of nodes in the existing network increases.

Assume that there is a new peer with two links. When the new peer attaches to the existing network, two peers need to be selected from the network. If the two selected peers in the network are connected each other, the clustering coefficient of the new peer becomes one, otherwise its value is zero. Thus, the probability that the clustering coefficient of the new peer becomes one is equal to the probability that randomly chosen two peers from the network are connected each other. Intuitively, the probability becomes very small as the number of nodes increases so that ACC decreases as the number of nodes in the network increases. For this reason, the random topology that we initially generated (before we apply the rewiring algorithm) has very small ACC.

• The improvement of ACC is in inverse proportional to the value of β .

From the Fig. 1(a), in order for the peer A to improve its clustering coefficient by the rewiring process, the final destination peer G needs to have more connections to the neighbor peers of the peer A than the peer E has. Thus, if the peer G has more links than the peer E, the peer G has high probability to have more connections to the neighbor peers of peer A.

Since a neighbor peer of a randomly chosen peer tends to be a large degree peer, the peer E that is a neighbor peer of the randomly chosen peer A tends to have more degrees than the peer G. Thus, rewiring the network with the first case only ($\beta = 1.0$) does not improve its ACC much. On the other hands, if rewiring is done to the network with the second case only ($\beta =$ 0.0), the peer B tends to have more links than the peer A since the peer B is a neighbor peer of the randomly chosen peer A so that we see a large improvement on ACC of the network. Finally, we can achieve medium values of ACC from the combination of the first and the second cases $(0.0 < \beta < 1.0)$. As mentioned previously, the clustering coefficient of a network is known to limit the spread of malicious code [9] so that this topological property can be considered to construct more robust network, e.g., slow down virus spreading.

5.1.2 Average shortest path (ASP)

• ASP is reduced as the value of β decreases.

The peer A and G in Fig. 1(a), and peer B and E in Fig. 1(b) have different degree characteristic. Keeping in mind that a neighbor peer of a randomly chosen peer tends to have large degrees, let us consider which peer loses a link and which peer gains the link.

In the first case, the neighbor peer E of the randomly chosen peer A loses a link, and the link is connected to the peer G. Since we do not know the degree state of peer G, we can only say that a large degree peer keeps losing a link and it moves to a random peer when the first case is used for rewiring.

On the other hand, in the second case, the randomly chosen peer A loses a link and the link is added to the neighbor peer B of the randomly chosen peer A. In other words, a large degree peer keeps gaining a link that belongs to a randomly chosen peer.

In summary, when a network is rewired using the first case, a high degree peer loses a link, and the link is used to connect two randomly chosen peers. For the second case, a randomly chosen peer loses a link, and the link connects two high degree peers. From this observation, we can conjecture that the second case constructs a network that has many large degree peers so that small degree peers connected to the large degree peers can be reached each other through the large degree peers within small number of hop counts. Thus, ASP decreases more when a network is rewired using the second case which implies rewiring with a small value of β .

5.2 Comparison of the proposed rewiring method with Watts & Strogatz (WS) model

In this section, we compare the proposed method with another well known rewiring method called WS model that was introduced by Watts & Strogatz in [12]. The reasons to choose this model for this comparison study are; Firstly, the model is based on a rewiring approach that is same as our proposed algorithm. Secondly, the idea of the rewiring in WS model was applied to a structured P2P network construction algorithm called Chord [1]. The WS model is referred as a small world model which means that a small diameter topology can be obtained using this model. A small diameter topology is desired especially for unstructured P2P networks where searching a peer that possesses desired files or data is the responsibility of individual peers. It is because a desired peer tends to be found easily in a small diameter network.

Since the performance of a topology can be evaluated in terms of search efficiency (How fast a peer can search for desired data from the network.), two searching algorithms, namely flooding and random walk searches, are implemented on topologies from the proposed rewiring algorithm and WS model[†], and their performances are analyzed.



Fig. 5 Network coverage ratios by flooding search (FDS) and random walk search (RWS) in topologies that are obtained by the proposed algorithm (Solid lines, each line represents the result with a different value of β) and WS model (Dot line).

Flooding search (FDS) is the most well known searching mechanism in unstructured P2P networks [16]. A message is sent by a peer to its all adjacent nodes, and the nodes that receive the message resend it to its nearest neighbor nodes excluding the source node. This process is replicated within a certain number of times called time-to-live (TTL). On the other hands, Random Walk search (RWS) has been used as an alternative search mechanism since FDS generates very large amount of messaging traffic as well as poor granularity, i.e., one additional step (TTL) significantly increases the total messaging traffic in a network [16]. When a peer searches for a peer that holds a desired file using RWS, it sends a message to one of its neighbors, and this process is repeated until the message is passed on to a neighbor of the target.

Fig. 5 shows the efficiencies of FDS and RWS on

[†]Since WS model[12] produces the smallest diameter when the rewiring probability p is equal to 1, we make use of the value for a fair comparison.

two different types of topologies. Topologies from the proposed method with different values of β are represented as solid lines, and the optimized WS topology is shown as a dot line. There are two interesting observations. Firstly, all topologies from the proposed rewiring method provide higher searching efficiency than the WS topology. For instance, the worst case topology of the proposed method (when $\beta = 1.0$) has nearly twice higher coverage ratio than the one from WS model in RWS at 500 TTL. Secondly, a topology from the proposed method with small β achieves higher searching efficiency than the one with large β . Actually, this result can be expected from the analysis of the basic topological properties shown in Fig. 4. It is because a small diameter topology tends to provide good search efficiency.

Other than this searching efficiency comparison between the proposed method and WS model, another noticeable difference is that the proposed method is able to manipulate the clustering coefficient of a network while WS model is not. A topology with higher clustering coefficient can handle heavy traffic more efficiently [18]. As shown previously in Fig. 4, the proposed method can transform a topology with nearly no clustering coefficient into a topology with different levels of clustering coefficient.

5.3 A scenario for the usage of the proposed rewiring method

We showed previously how the proposed rewiring method affects the topological properties in Fig. 4 as well as the network performance in Fig. 5.

With the observation, we can use the proposed method to change the topological properties or to boost the performance of the network in a self organizing manner when it is necessary. For instance, under the assumption that the search efficiency of the network is monitored (e.g., probe packet), when it becomes less than a threshold value (low search efficiency)^{\dagger}, the proposed rewiring method can be triggered to boost the search efficiency of the network. In this example, the monitoring system triggers the event and is responsible for selecting random peers to carry out the rewiring process. As described in Section. 3, the random peers can be obtained from the dedicated hardware that maintains the identifications of existing peers in the networks, or from some algorithms such as a random walk approach proposed by Vishnumurthy et al [15].

Regarding to the choice of the value β , when the size of the network N is equal to 10^4 , the result shown in Fig. 5 can be used to select a suitable β to raise the search efficiency over the threshold value. For instance, assuming that the threshold value of the coverage ratio

is 0.7 at 500 TTL when random walk search (RWS) is used in the network, the value of β needs to be set less than or equal to 0.3 in case that the coverage ratio becomes less than the threshold value.

5.4 Convergence of the proposed method

Although, we analytically showed the emergence of a certain topological structure through the proposed rewiring method in Section 4, it is interesting to find out how long it takes for the topological transformation. We initially constructed ER random topologies that have different number of nodes and average degrees, and then observe the variations of degree distributions as the number of rewiring times per individual peers increases. To observe the evolution of the degree



Fig. 6 The variation of Kullback distance between the degree distributions of the final topology and that of the topology whose links are rewired continuously. Top figure: initial ER random topologies have N=5000 nodes and the average degrees $\langle k \rangle$ are $(10 (\circ), 14 (\triangleleft), 20 (\diamond))$, Bottom figure: initial ER random topologies have the average degree $\langle k \rangle$ of 10 and the sizes of networks N are $1000 (\circ), 3000 (\triangleleft), 10000 (\diamond)$.

distribution through the rewiring process, we make use of Kullback distance.

The Kullback distance measures how much one probability distribution is different from the other probability distribution. Since the rewiring process changes the degree distribution of the network, the change in the degree distribution can be quantified by measuring the Kullback distance between the two degree distributions which are before and after rewiring is done on the network. For instance, when two degree distributions are $P(k)^{t_1}$ and $P(k)^{t_2}$ at time t_1 and t_2 , the Kullback distance between them is defined in equation (12).

$$K(P(k)^{t_1}, P(k)^{t_2}) = \sum_{k=n}^{M} \sum_{k=n}^{M} P(k)^{t_1} \log \frac{P(k)^{t_1}}{P(k)^{t_2}}$$

[†]We do not consider which value is sufficient for P2P networks since it depends on the network operation.

Fig. 6 shows that the degree distribution of the topology converges as the number of rewiring times per peer increases. Comparing the top and bottom figures, the average degree of the topology seems to affect the convergence time more than the size of the network does. Additionally, the convergence time is delayed as the average degree of the topology increases.

5.5 A truncated power law topology

In unstructured P2P networks, traffic loads need to be distributed among peers rather than centralized on a few of them. Due to the reason, there has been a criticism that a power law topology may not be suitable for the construction of P2P network topology since a few hub peers in a power law network suffer from a huge amount of data. Thus, it is desirable to limit the maximum number of available links per node. As mentioned previously in Section 2.1, a power law topology whose peers are restricted to have maximum number of links is called a truncated power law topology.

Guclu et al [16] proposed a method to construct a truncated power law topology, and showed how P2P networks can benefit from the truncated power law topology. However, their approach is a topology construction method which focuses on how newly appeared peers join to the existing network rather than how individual peers are rewired. Thus, we demonstrate here that a truncated power law topology can be also obtained from our proposed rewiring method.

The parameters M and n are used to limit the number of maximum and minimum degrees that individual peers are allowed to have. Let the equation (8) be substituted into the equations (9) and (10), and cancelling P(n) from the both equations give

$$\sum_{k=n}^{M} n(\frac{\langle k \rangle}{k} - 1)(\frac{a}{b})^{k-n} = 0$$
(13)

By substituting $a/b=1^{\dagger}$ into equation (13), we finally obtain equation (14) which shows how the given two parameters M and n of the proposed rewiring model need to be set in order to transform a topology into a power law topology.

$$\langle k \rangle = \frac{M-n+1}{\sum_{k=n}^{M} 1/k}$$
 (14)

Interestingly, the equation (14) does not include the size of network N as a variable. In other words, the rewiring method can transform topologies with different size into the same power law topology as long as their average degrees are same.

To verify the condition shown in equation (14), supposed that there is a topology with the average degree $\langle k \rangle$ of 10. By varying the parameter n from 1 to 5, the counterpart values of M become 44, 31, 25, 21, and 18 according to equation (14). Top five figures in Fig. 7 show the emergence of the truncated power law topologies (lines with dots on them). All truncated power law topologies have the exponent values of -1. Down five figures show the emergence of almost identical power law topologies regardless of the size of original topologies. As mentioned previously, it is because equation (14) does not include the size of network N as a variable.



Fig. 7 Degree distributions of Erdos and Renyi (ER) random topologies are plotted with lines with crosses on them (Top five figures: <k>=10, $N=10^4$, Down five figures: <k>=10, N=2000, 4000, 6000, 8000, 10000), and these topologies are rewired using the proposed method (Top five figures from left to right: (n=1, M=44), (n=2, M=31), (n=3, M=25), (n=4, M=21), (n=5, M=18), Down five figures: (n=1, M=44)) and their degree distributions are plotted with lines with dots on them.

6. Conclusions

A rewiring method using one parameter has been proposed. By tuning the parameter, a network topology can be changed to have different topological properties that are closely related to the performance of networks. The only rule that each peer follows is to pass one of its neighbor peers' ID on request. This simplicity enables the proposed method to transform the network topology in a self organizing manner so that the network adopting this method can inherit various advantages of a self organizing system such as scalability, security, and robustness.

The evaluation study showed that a small diameter and highly clustered network can be emerged through

(12)

[†]We notice that the equation (8) represents a power law distribution with the exponent γ of -1 when a/b is equal to one.

the proposed rewiring method. Based on the observation, we demonstrate how P2P networks can take advantage of the emerged topological property in terms of the analysis of searching efficiency. In addition, we demonstrated that the proposed rewiring method can transform a topology into a truncated power law topology.

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