On the Packet Delay Distribution in Power-law Networks

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***Abstract*—Measurement studies of the Internet topology have revealed that the degree distribution exhibits a power-law attribute. That is, the probability $P(k)$ that a node has k out-going links follows $P(k) \sim k^{-\gamma}$. However, it is known that the power-law degree distribution alone does not determine traffic-level behaviors in Internet topologies. In this paper, we investigate packet-level delay behavior of topologies having power-law degree distribution. Our results show that the packet delay distribution of the actual ISP topology also follows the power-law, while the delay distribution of model-based topology does not. We then investigate the structural differences between the topologies, and show that the modularity structure of ISP's router-level topologies makes the packet delay distribution being long-tail.**

***Index Terms*—Power-law networks, End-to-end flow control, Packet delay, ISP's router-level topologies, BA model**

I. Introduction

Measurement studies of the Internet topologies have revealed that the degree distribution of the topologies exhibits a power-law attribute. That is, the probability $P(k)$ that a node has k out-going links is proportional to $k^{-\gamma}$. In such topologies, lots of nodes are connected with a small number of nodes, and a few "hub" nodes are connected with a large number of nodes.

While the origin of this phenomenon is not clear formally, Barabási and Albert introduced the well-known BA model to generate topologies having power-law degree distributions [1]. The BA model has simple two rules to generate; incremental growth of nodes, and preferential attachment with respect to degrees of existing nodes. The resulting topologies of the BA model have low-diameter networks. Many researches investigate topological properties appeared by the BA model [2]–[6].

Some papers investigate traffic-level behaviors in topologies having power-law degree distributions [7]–[9]. In Ref. [7], the authors investigate the distribution of numbers of node-pairs that pass through a node with a BA-based generation model. They show that, with the minimum-hop packet routing, the load distribution of nodes also exhibits a power-law attribute. Reference [8] investigates how congestion of traffic propagates over topologies. In the paper, each router has a finite size buffer and has a flow control mechanism between routers. When the buffer of a router is fully occupied by packets, the upstream router stops sending packets to the congested router and waits for a congestion elimination. Reference [8] demonstrates that the congestion spreads easily over the topologies in the BA topologies because of the low-diameter of topologies. The low-diameter effects also appear in the queuing delay distribution of topologies. The paper shows that the queuing delay distribution of the BA topology follows a power-law when packet generating rate is low, while it follows a Pareto distribution as the packet generating rate gets higher. The effect of end-to-end flow control is also investigated on the topology obtained by the BA model [9]. The authors examined TCP control with long range dependence (LRD) input traffic and Poisson input traffic, and revealed that average of end-to-end packet delay sharply increases for both input traffic since packets more concentrate on the hub nodes in the BA topology.

In these papers, researchers use topologies generated by the BA model or its variant models. However, when router-level topologies are concerned, the BA model in which links are attached based on a preferential probability does not adequately model the structure of the ISP router-level topologies [10], since each ISP constructs its own router-level topology based on strategies such as minimizing the mileage of links and/or maximizing reliability [11], [12]. A failure modeling of structure of topology results in the failure of networking protocols; for example, Ref. [10] demonstrates that the link utilization of the ISP topologies is much lower than that of the BA topology. These papers clearly indicate that the power-law degree distribution alone does not determine traffic-level behaviors in router-level topologies. Thus, the investigation of Refs. [7]–[9] may not be applied to the ISP router-level topologies.

In previous studies, it has revealed that end-to-end flow control like TCP have large impacts on the traffic dynamics [13], [14]. However, these researches deal with simple small topologies, so, how structure of complex large topologies impacts on traffic dynamics is not clear. In this paper, we investigate packet-level behaviors in the ISP router-level topology where the degree distribution exhibits power-law attribute and each of nodes has end-to-end flow control functionality. Comparing results with the BA topology and the ISP topology, we discuss how structure of topologies and flow controls differs the end-to-end packet delay distribution. We use a stop-and-wait protocol for the end-to-end flow control instead of TCP protocols having various functions, such as slow-start and congestion avoidance, mainly because it is difficult to distinguish effects of TCP functions [15]. Results of our simulations show the packet delay distribution exhibit a long-tail distribution on the actual ISP router-level topology, while packet delay distribution of the BA topology does not. To explain this, we compare the structural differences between the model-based topology and the ISP router-level topologies, and show that the modularity structure of the ISP router-level topologies makes the packet delay distribution being long-tail.

This paper is organized as follows. In Section II we explain about the network model we examined. Section III shows the simulation results. Section IV conclude this paper and explain our future works.

II. Network Model

In this section, we explain about the network model used in this paper.

A. Network Topologies

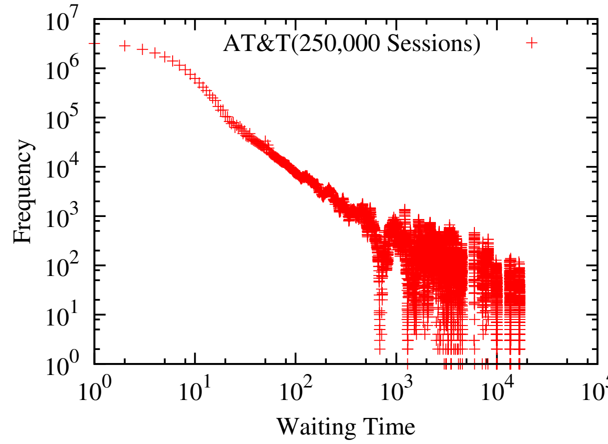
We use the AT&T topology measured by Rocketfuel tool [16] as ISP router-level topology. The topology has 523 nodes and 1304 links, and the degree distribution of the AT&T topology follows a power-law [10]. For comparison purpose, we also use the BA AT&T topology generated by the BA model. The BA AT&T topology is generated such that the numbers of nodes and links of it are the same as that of the AT&T topology.

B. Node Processing Model

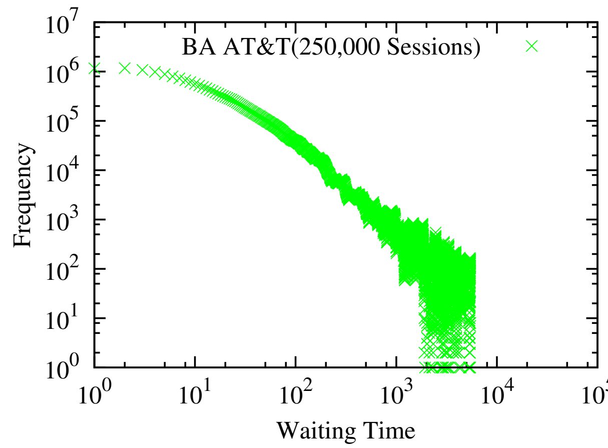
Each node has infinite buffers at each out-going links. When a packet arrives at a given node and when the node is the packet's destination, the node removes the packet from the network. Otherwise, the node selects the next node based on a minimum hop routing algorithm, and forwards the packet to a buffer of an out-going link connecting to the next node. Each outgoing link sends packets to the next node based on FIFO queuing discipline, and delivers one packet per unit of time. Here, we do not use the dynamic routing, i.e., each packet traverses the shortest path calculated beforehand. If multiple shortest paths are found, the next node is selected randomly.

C. Flow Control between Nodes

Before starting the simulation, pre-specified numbers of sessions are created between nodes. For each session, source and destination nodes are randomly selected. Each session sends packets based on a stop-and-wait protocol. That is, when a source node sends a packet to its destination node, the source node stops sending a new packet until the source node receives the ACK packet from the destination node. Since we want to investigate end-to-end packet delay distributions on topologies having different structure, we do not consider the packet loss inside the network and do not consider time-out operations at the source nodes. By our protocol, once the source node sends a packet, the source node always obtains the ACK packet, though it may take a long time. Actually, this protocol does not reflect the packet delay in the Internet, but the protocol does reflect the differences of packet delay distribution over the topologies more clearly.



(a) AT&T Topology



(b) BA AT&T Topology

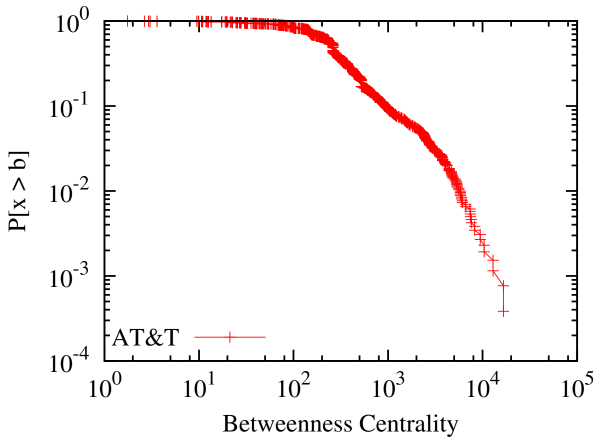
Fig. 1. Waiting time distribution on two topologies. X-axis represents waiting time in the queue, and Y-axis represents frequency of X-value. The number of sessions is 250,000. Simulation time T is 100,000. Data is gathered from T = 90,001 to T = 100,000.

III. Simulation Results and Discussions

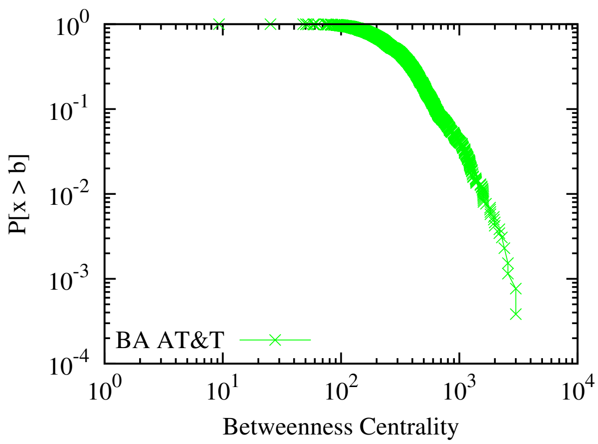
In this section, we simulate the traffic flow on the AT&T topology and the BA AT&T topology using the network model explained in the previous section. We run each simulation for 100,000 unit of time, and collected the packet delay appeared in the last 10,000 unit of time. We use original simulator written in C++.

A. Waiting Time Distribution

Figure 1 shows the distribution of waiting time at out-going links on the AT&T topology and the BA AT&T topology. The waiting time is the time from when a packet is stored in a

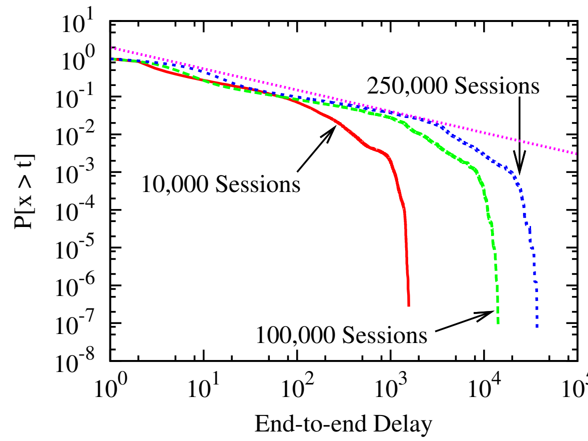


(a) AT&T Topology

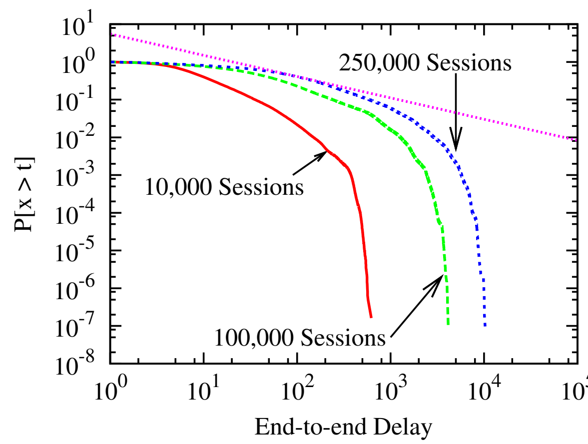


(b) BA AT&T Topology

Fig. 2. Betweenness centrality distribution (CCDF): X-axis represents betweenness centrality and Y-axis represents existing probablity of links which has larger centrality than X-value.Each distribution follows a power-law.



(a) AT&T Topology



(b) BA AT&T Topology

Fig. 3. Packet delay distribution (CCDF): On AT&T topology, as the number of sessions gets larger, a long-tail distribution arises clearly. On the other hand, a long-tail distribution does not appear on the BA AT&T topology. We also show $P[x > t] \sim x^{-0.56}$ (straight lines in these figures).

buffer to when the packet is transfered to the next hop. Here, the waiting time is equivalent to the number of packets in the queue since it is assumed that all links deliver one packet per one unit of time. In the figure, X-axis represents waiting time in FIFO queues of out-going links, and Y-axis represents frequency of the X-axis value.

Figure 1 shows that the distribution of waiting time follows a power-law in both topologies. The main reason for this is the distribution of link betweenness centrality. The link betweenness centrality is defined for each link as the fraction of shortest paths that passes through the link, counted over all pairs of nodes.

Figure 2 shows distribution of link betweenness centrality in the AT&T topology and the BA AT&T topology. X-axis represents betweenness centrality and Y-axis represents the complementaly cumulative distribution function of betweenness centrality. We observe from this figure that the distribution of betweennes centrality of links also follows a power-law. Given large number of sessions inside the network, the number of packets that pass through the link is proportional to the betweenness centrality of the link. As a result, the waiting time distribution also exhibit power-law attribute.

B. End-to-End Packet Delay Distribution

Figure 3 shows end-to-end packet delay distribution of the AT&T topology and the BA AT&T topology. X-axis represents the packet delay and Y-axis represents the complementary cumulative distribution function of the packet delay. Here, the end-to-end packet delay is the time from when a packet is generated at source nodes to when the packet arrives at its destination nodes.

In either topologies, as the number of sessions gets higher (from 10,000 sessions to 250,000 sessions), packets spend more time in the network since the load of network gets heavier. The important point is that the shape of packet delay distribution of the AT&T topology is much different from the shape of packet delay distribution of the BA AT&T topology, especially when the number of sessions is large. The

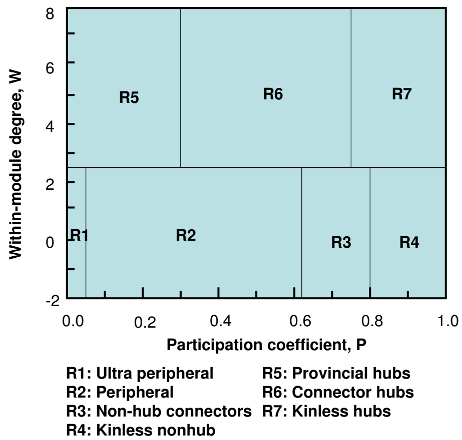
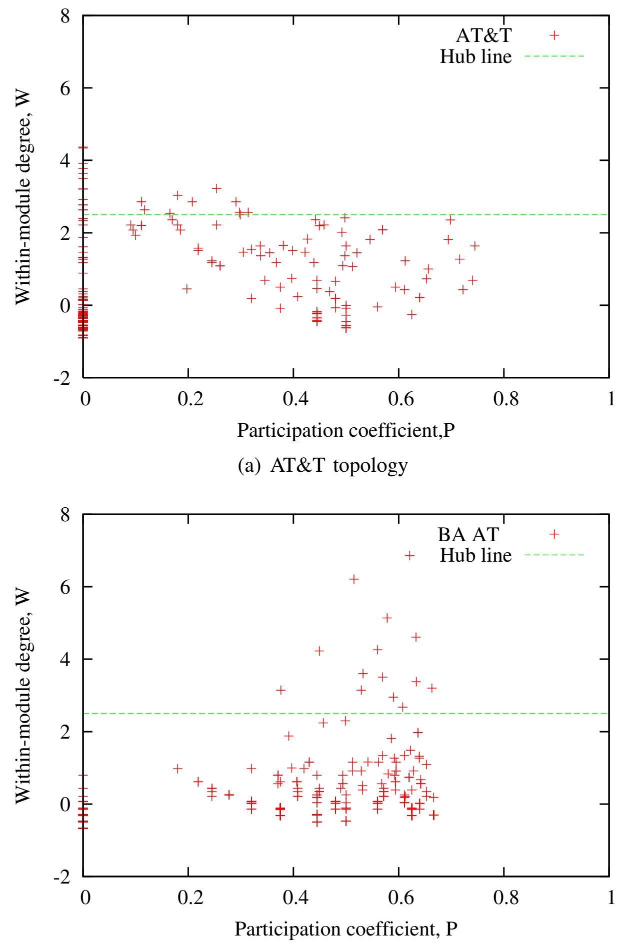


Fig. 4. Classification of node function with participation coefficient and within module degree



(a) AT&T topology

(b) BA AT&T topology

Fig. 5. Classification of node function in each topology

packet delay distribution of the AT&T topology has a long-tail distribution; that is, the distribution is characterized by the slow decay at the larger packet delay. (Fig. 3(a)). However, the packet delay distribution of the BA AT&T topology does not show the long-tail distribution. (Fig. 3(b)).

These results indicate that the distribution of end-to-end packet delay differs dependent on the topology, more precisely the structure of topology. The next section discusses what a structure of topologies makes the delay distribution to be power-law.

C. Effects of Structure of Router-level Topology

In the Section III-B, we show that the end-to-end packet delay distributions of two topologies exhibit different attributes though two topologies have the similar shape of the waiting time distributions. In this section, we compare the structural differences of the AT&T topology and the BA AT&T topology. As discussed in Ref. [12], design principles of networks greatly affect the structure of the ISP topologies. Design principles determine a node functionality, which in turn determines the connectivity of nodes.

In [17], Guimeara et al. have proposed the classification method of node functions. The method divides a network to multiple modules and defines the within-module degree Z_i, and the participation-coefficient, P_i, for each node i. Assuming that the node i belongs to a module s_i, the within-module degree Z_i of node i is defined as,

$$Z_i = \frac{k_i - < k_{s_i} >}{\sigma_{s_i}}, \quad (1)$$

where k_i is the degree of nodes, $< k_{s_i} >$ represents the average degree in module s_i, and σ_{s_i} is the variance of the degree distribution of nodes in module s_i. The participation-coefficient P_i of node i is also defined as,

$$P_i = 1 - \sum_{s=1}^{N_m} (\frac{k_{is}}{k_i}), \quad (2)$$

where k_{is} represents the fraction of links connecting with the module s_i. That is, when all the links of node i connect with nodes belonging to the same module of s_i, P_i becomes 0.

Figure 4 shows the roles of nodes are categorized by the value of Z_i and P_i, and Figure 5 shows the result of application of the Guimeara's method to the AT&T to pology and the BA AT&T topology. The module is calculated from the method in [18]. In Figs. 4 and 5, the horizontal axis indicates within-module degree Z and the vertical axis the participation coefficient P. Depending on the values of P and Z, the role of node is categorized into several classes. For example, when Z_i is large and P_i is relatively large, the node i has many links connecting to other modules. Thus, the node i is categorized into the "Connector hub(s)". "Provintial hub(s)" also takes the larger Z_i but smaller P_i; the node i has many links connecting with nodes in the same module.

Looking at Fig. 5, the BA AT&T topology has many "Connector hub" nodes that transfer large amount of packets between modules. However, Fig. 5(a) shows that there are no "Connector hub" nodes in the AT&T topology. This means that the AT&T topology has a few inter-module links. In the

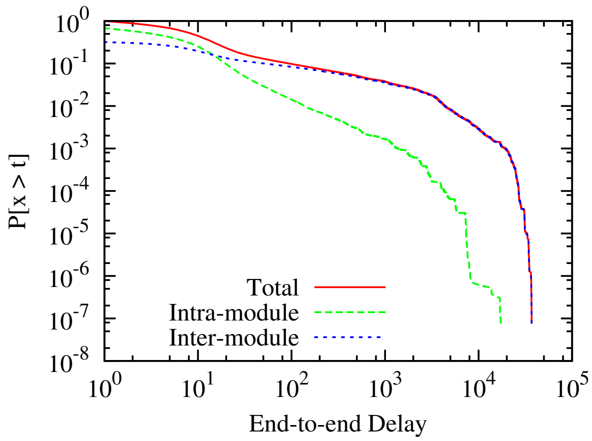


Fig. 6. Comparison of packet delay distribution of intra-module packets and inter-module packets: The number of sessions is 250,000, and data is gathered from T = 90,001 to T 100,000. Long-tail distribution is caused by inter-module packets.

AT&T topology, packets traveling between modules are first aggregated at "Provincial hub" nodes and then forwarded via "Non-hub connecter" nodes. Thus, inter-module links in the AT&T topology tend to be congested, and packets passing through the links experience a long end-to-end delay.

To see the impact of the modularity structure of the AT&T topology, we separate the packet delay distribution into inter-module packets, where packets traverse through the inter-module links, and intra-module packets, where packets traverse only the intra-module links. Figure 6 shows the complementary cumulative distribution of packet delay for intra-module packets and inter-module packets. Looking at the packet delay distribution of intra-module packets, we notice that most of packets arrive at destination nodes within a short time and the probability taking larger packet delay decays drastically. By contrast, the packet delay distribution of inter-module packets exhibits long-tail characteristic; the probability taking larger packet delay does not decrease so fast when we compare it with the results for intra-module packets. The modularity structure of the AT&T topology makes the inter-module links to be congested, which leads to the long-tailed packet delay distributions.

IV. Concluding remarks

In this paper, we evaluated the packet-level behavior on the AT&T topology and the BA AT&T topology having power-law degree distribution. Our simulation results show that the packet delay distribution of the AT&T topology exhibits a long-tail attribute, while the distribution of the BA AT&T topology does not. We then investigated how structural property of topology affects on the packet delay distribution. The main reason causing the long-tail distribution is the modularity structure of the AT&T topology; the inter-module links make the packet delay being long. To prevent the packet delay distribution from being long-tail, one approach is to construct a topology with many inter-module links like the BA AT&T topology so that congestion does not occur on the inter-module links. Another approach is to assign the link bandwidth properly. Our next topic is to consider the optimal design of topology from flow-control perspective. For this purpose, we will conduct evaluations of packet delay distribution on topologies that have heterogeneous link capacity, and evaluation of combination of flow-control between routers and end-host flow control in more detail.

Acknowledgement

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