PAPER

Analyzing and Utilizing the Collaboration Structure for Reliable Router-Level Networks

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SUMMARY As the Internet represents a key social infrastructure, its reliability is essential if we are to survive failures. Physical connectivity of networks is also essential as it characterizes reliability. There are collaboration structures, which are topological structures where two or more nodes are connected to a node, and collaboration structures are observed in transcriptional regulatory networks and the router-level topologies of ISPs. These collaboration structures are related to the reliability of networks. The main objective of this research is to find whether an increase in collaboration structures would improve reliability or not. We first categorize the topology into a three-level hierarchy for this purpose, i.e., top-level, middle-level, and bottom-level layers. We then calculate the reliability of networks. The results indicate that the reliability of most transcriptional regulatory networks is higher than that of one of router-level topologies. We then investigate the number of collaboration structures. It is apparent that there are much fewer collaboration structures between top-level nodes and middle-level nodes in router-level topologies. Finally, we confirm that the reliability of router-level topologies can be improved by rewiring to increase the collaboration structures between top-level and middle-level nodes.

key words: *network reliability, transcriptional regulatory network, router-level topology, collaboration structure, power-law network*

1. Introduction

As the Internet has become a social and economic infrastructure, its reliability is essential if we are to survive failures. Many approaches to improving its reliability have been investigated either at the network layer [1] or higher layers [2] in OSI model. The reliability of optical communication systems has also been improved through protection/restoration techniques [3].

While these approaches have greatly improved the reliability of networks, physical connectivity of networks is more essential to characterize their reliability. That is, if physical connectivity of networks is easily disrupted by network failures, approaches to improving reliability at the network layer will no longer be effective. In fact, the physical topologies used in the previous studies have inherently assumed that physical connectivity is retained after network failures occur. It is important to make the physical topology reliable against network failures to design reliable networks. It is also necessary to investigate the topological characteristics and topological structures that make the physical topology more reliable to achieve this purpose.

Regular topologies have also been studied to construct reliable networks. One example is a hypercube structure [4] where each node has an identical number of out-going links that are interconnected through a regulated wiring rule. Failure-tolerant characteristics of regular topologies have recently been intensively studied to enhance the reliability of data center networks [5], [6]. However, unlike regular topologies, the degree distribution of router-level topologies of ISPs on the Internet exhibits power-law attributes, meaning that the existing probability, $P(k)$, of a degree k node that has k links is proportional to $k^{-\gamma}$ [7]. This means that we have to make drastic changes to the topology from the current router-level topology to benefit from the failure-tolerant characteristics of these regular topologies, which is an unrealistic approach to enhancing reliability.

The main objective of this research is to investigate topological structures that should be embedded to make router-level topologies more reliable on the basis of knowledge in biological systems. More precisely, we evaluate the topological structure of a transcriptional regulatory network for several species that have a much longer evolutional history than information networks, and investigated what effect introducing its topological structure into router-level topologies would have.

Transcriptional regulatory networks are biological system where transcription factors regulate the genes in cells, and control the expression of genes to produce the proteins necessary for biological activities [8], [9]. The degree distribution of these networks also exhibits power-law attributes like router-level topologies [10]. Balaji et al. [8] explains that transcriptional regulatory networks have many collaboration structures where two or more transcription factors co-regulate other transcription factors (see the definition in Sect. 2.2). The collaboration structures contribute to making the topologies reliable because they introduce multiple paths between nodes, and are therefore generally more reliable against failures in transcription factors. As we will see in Sect. 2, connectivity after multiple failures in *E. coli*, taking an average degree of 1.55, is higher than that in an ISP router-level topology, taking an average degree of 1.87. That is, the transcriptional regulatory network is more reliable than the ISP router-level topology. Bhardwaj et al. [9] classified nodes into top-level, middle-level, and bottom-level layers of a hierarchy, and they investigated the degree of collaboration between these three layers. Their results

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indicated that the transcription factors of the middle level are co-regulated the most, and complex organisms like humans collaborate more than other organisms such as *E. coli* or yeast. The results obtained by Balaji et al. and Bhardwaj et al. [8], [9] are significant from the biological point of view. However, our interest here is the reliability of router-level networks. That is, it is important to analyze the difference in the collaboration structures between router-level topologies and transcriptional regulatory networks.

We investigated topological structures that made router-level topologies more reliable based on an analysis of transcriptional regulatory networks with collaboration structures, which is discussed in this paper. We particularly focused our attention on collaboration structures related to robustness and analyzed the difference in collaboration structures between router-level topologies and transcriptional regulatory networks by using comparative investigations. We first investigated whether the router-level topologies of ISPs had already obtained the topological structures that appeared in living organisms. As we will see in Sect. 3, there is a clear difference in the collaboration structures between transcriptional regulatory networks and router-level topologies; there are much fewer collaboration structures between top-level and middle-level nodes in router-level topologies. To check what effect such structures had on reliability, we examined rewiring to increase the collaboration between top-level and middle-level nodes in router-level topologies, and observed the differences in reliability before and after rewiring was carried out. Note that we did not intend to actually rewire the links in router-level topologies. Rewiring did not retain the number of links in the physical topology, but changed the topological structures of router-level topologies.

This paper is organized as follows. Section 2 describes transcriptional regulatory networks and similarities between the networks and router-level topologies. Section 3 presents a definition of collaboration structures in biological networks and router-level topologies. We evaluated the number of collaboration structures by using a metric called the degree of collaboration, which is explained in Sect. 4. We then investigated the effects of collaboration structures on reliability by changing the physical topology through the rewiring process explained in Sect. 5. Finally, we conclude the paper in Sect. 6.

2. Reliability of Transcriptional Regulatory Networks and Router-Level Topologies

2.1 Analogies between Transcriptional Regulatory Networks and Router-Level Topologies

Transcriptional regulatory networks represent biological systems where transcription factors regulate the genes in cells, and control their expression. Each gene generates its corresponding protein, which is necessary for biological activity in cells. Transcription factors in transcriptional regulatory networks are collaborated each other and co-regulate other transcription factors or genes.

There are various analogies between transcriptional regulatory networks and router-level topologies. For example, the degree distributions of both networks exhibit power-low attributes. Another similarity is their hierarchical structures. There are three levels of hierarchy in transcriptional regulatory networks, i.e., top-level, middle-level, and bottom-level layers [9]. Router-level topologies also have a hierarchy in a network, e.g., a core network is connected with several regions and/or states, regional networks, and access networks. The collaboration structures in transcriptional regulatory networks correspond to load balancing and/or alternate paths in router-level topologies. That is, the collaboration structures contribute to the reliability of topologies because they introduce multiple paths between nodes, and are therefore generally more reliable against failures in transcription factors.

We evaluate the reliability of transcriptional regulatory networks and router-level topologies, which are discussed in the following subsection. We also investigate and analyze the hierarchies and collaboration in router-level topologies and transcriptional regulatory networks, which are explained in Sect. 3.

2.2 Reliability

In this section, we compare the reliability of router-level topologies and the transcriptional regulatory networks. Note that transcriptional regulatory networks are directed networks, and router-level topologies are undirected networks. Nevertheless, the reliability of both transcriptional regulatory networks and router-level topologies should be evaluated by the same measure. Therefore, we replace undirected links in router-level topologies to directed links by following procedures.

Since the traffic is usually aggregated at a regional network and then forwarded to the backbone networks in router-level topologies, the backbone network is located at the “center” (in terms of hop-counts) of network and the top-level nodes defined by modularity analysis are backbone routers. In addition, nodes that are apart from “center” of network are regarded as bottom-level nodes because these nodes do not relay the traffic. Thus, our approach to define the direction of links is valid under the condition that router-level topologies aggregate the traffic at their backbone network. We suspect that our approach does not work when the router-level topologies do not have a hierarchical structure and traffic aggregation is not intended. However, we believe that such the situation merely occurs in the router-level topologies, and we actually observe that the hierarchical structure and the traffic aggregation by looking at Fig. 6 and Fig. 9 of Ref. [11].

In the transcriptional regulatory networks, top-level nodes receive stimuli from the external environment. For the router-level topologies, we regard the stimuli as the traffic from top-level nodes to bottom-level nodes. We therefore introduce the reachable node ratio for investigating reliabil-

Table 1 Numbers of nodes and links in *E.coli*, human, mouse, rat, and yeast transcriptional regulatory networks.

	E.coli	Human	Mouse	Rat	Yeast
Nodes	80	88	78	30	127
Links	124	327	160	39	421
Links/Nodes	1.55	3.72	2.05	1.3	3.31

Table 2 Numbers of nodes and links in eight router-level topologies of AT&T, Ebone, Exodus, Level3, Sprint, Telstra, Tiscali, and Verio.

	AT&T	Ebone	Exodus	Level3	Sprint	Telstra	Tiscali	Verio
Nodes	523	140	157	623	467	329	240	839
Links	1304	261	283	5298	1280	616	403	1889
Links/Nodes	2.49	1.86	1.80	8.50	2.74	1.87	1.68	2.25

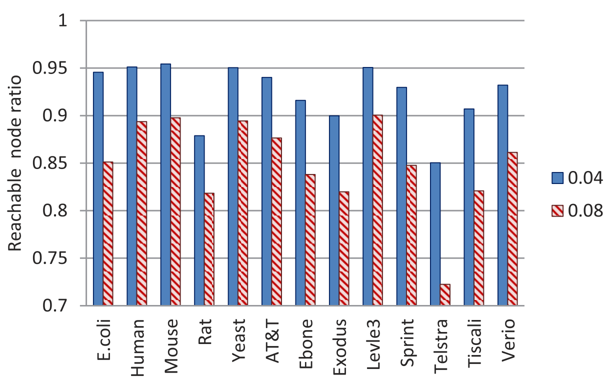

Fig. 1 Ratio of reachable nodes from top-level nodes where failure node ratios are 0.04 and 0.08. Most transcriptional regulatory networks are more reliable than router-level topologies. *E.coli* and rat networks are less reliable than other transcriptional regulatory networks because link density of these two organisms networks is lower.

ity of directed networks, and evaluate the number of nodes that receive stimuli or traffic from top-level nodes after node failures.

We consider the random node failures in each network, and we evaluate the ratio of nodes that can be reached from top-level nodes to the number of nodes in the network. After this, we will call the ratio the *reachable node ratio*. We use the transcriptional regulatory networks of five species, i.e., *E.coli*, human, mouse, rat, and yeast [9]. The original data on transcriptional regulatory networks in Bhardwaj et al. [9] does not guarantee connectivity between any nodes. We have only considered the largest connected components to compare transcriptional regulatory networks with router-level topologies in this paper. The numbers of nodes and links for five transcriptional regulatory networks are summarized in Table 1. For purposes of comparison, we also use the eight router-level topologies of AT&T, Sprint, Ebone, Exodus, Level3, Telstra, Tiscal, and Verio [12]. These topologies are obtained from trace-route-based measurements of networks, which may require alias resolution. The rocketfuel in Ref. [12] extended the Mercator project's method [13] and relaxed the possibility of IP aliasing of routers to some extent. The numbers of nodes and links for eight router-level topologies are summarized in Table 2.

Figure 1 shows the reachable node ratio, which is dependent on the failure ratio. The failure ratio is defined as the number of failed nodes normalized by the number of nodes in the original network. Nodes to fail are selected randomly from a set of nodes in the top or middle levels to obtain the figure since bottom-level nodes are located at the edge of the network and removing them does not have an impact on the reachable node ratio. Figure 1 indicates the reachable node ratios when the failure ratios are 0.04 and 0.08. We can observe from this figure that human, mouse, and yeast transcriptional regulatory networks are the most reliable of the organisms that we investigate. As this figure shows, *E.coli* and rat networks are not more reliable than the other organisms, and even lower than some router-level topologies. Looking at Table 1, the reason for this is that the link density of *E.coli* and rat networks is much lower than that of other networks. When we compare the *E. coli* and Telstra networks whose average degrees are almost the same, the reachable node ratio for *E. coli* is higher than that for Telstra. This indicates that transcriptional regulatory networks are generally more reliable than router-level topologies.

We will focus on the collaboration structures of route-level topologies and investigate the difference in collaboration structures between router-level topologies and transcriptional regulatory networks from the beginning of the next section.

3. Collaboration in Networks

3.1 Collaboration in Biological Networks

The collaboration structure in transcriptional regulatory networks was investigated by Bhardwaj et al. [9]. The collaboration structure in transcriptional regulatory networks is a co-regulation relationship where two transcription factors regulate a transcription factor. According to the results obtained by Bhardwaj et al. [9], more complex organisms such as those of humans have more collaboration structures.

A key to identifying collaboration structures is to find a hierarchy, i.e., top, middle, and bottom levels in router-level topologies and transcriptional regulatory networks. We therefore investigate the collaboration structures in router-level topologies and find differences in the collaboration structures of router-level topologies and transcriptional regulatory networks. We then examine changes in the collaboration structures to discover future directions in designing a reliable router-level topology.

3.2 Definition of Hierarchy in Transcriptional Regulatory Networks

Top-level nodes in transcriptional regulatory networks do not have any incoming links, and middle-level nodes have both incoming links and outgoing links [9]. The other nodes are categorized into bottom-level nodes that are only regulated by other nodes.

3.3 Definition of Hierarchy in Router-Level Topologies

We define top, middle, and bottom-level nodes in router-level topologies as follows. Top-level nodes are determined through modularity analysis [14]. We divide the topologies into modules, and a node having one or more links that are connected with other modules is classified as a top-level node. Note that top-level nodes in transcriptional regulatory networks receive stimuli from the external environment. External stimuli can be regarded as traffic from other modules in the current case for router-level topologies.

We next calculate H_i as the average hop count from node i to other nodes. Then, we set a directed link from node i to node j when H_i is lower than H_j if undirected link (i, j) exists in the router-level topology. That is, when node i is located at the "center" of the network, the node tends to become a higher-level node. When the node is located at the "edge" of the network, the node tends to become a lower-level node. However, when there is a directed link toward the top-level node, we reverse the direction of the link so that we do not have links from the lower level layer to the top-level layer. When there is a directed link between top-level nodes, we change the directed link to become a bidirectional link. The link between a node pair whose nodes have the same average hop count is also regarded as being a bi-directional link. In this way, we construct a directional network from the router-level topology. Nodes in a directed network that have both incoming and outgoing links are classified into middle-level nodes, and nodes that only have incoming links are classified into bottom-level nodes.

3.4 Comparison of Hierarchical Structures in Transcriptional Regulatory Networks and Router-Level Topologies

We investigate the characteristics of the hierarchical structures of transcriptional regulatory networks and router-level topologies. Figure 2 shows the ratio of nodes in each level of hierarchy. We can observe that the number of bottom-level nodes is greater than the number of top-level or middle-level nodes in router-level topologies. In contrast, the ratio of middle-level nodes is large in transcriptional regulatory networks.

The ratio of links between levels of hierarchy is shown in Fig. 3. Transcriptional regulatory networks have numerous links between middle-level nodes but have few links from top-level nodes to bottom-level nodes. There are comparatively more links from top-level nodes to bottom-level nodes in router-level topologies. Since top-level nodes in transcriptional regulatory networks are not regulated by other top-level nodes, there is no link from top-level nodes to top-level nodes.

3.5 Definition of Collaboration

The collaboration structures in directed networks are structures where multiple higher-level nodes are connected with lower-level nodes. The collaboration structures contribute to the reliability of topologies because it introduces multiple paths between nodes, i.e., topologies that have many collaboration structures tend to be reliable. Here, we explain a metric, called the degree of collaboration, to compare it with the number of collaboration structures in topologies.

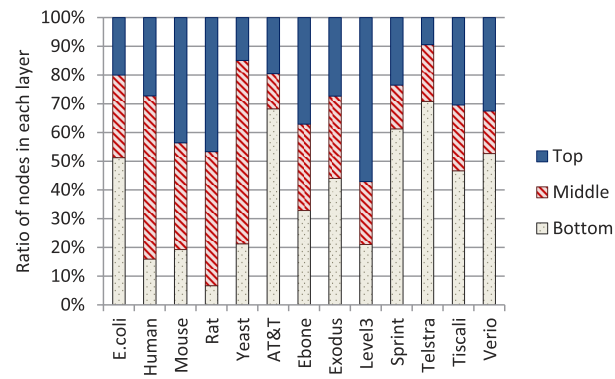

Fig. 2 Ratio of top-level, middle-level, and bottom-level nodes in each topology.

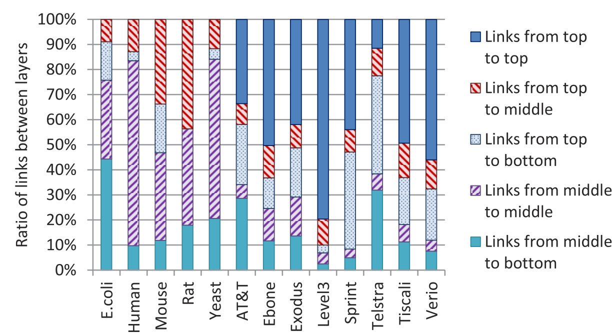

Fig. 3 Ratio of links between each level in hierarchy.

The degree of collaboration has been defined by Bhardwaj et al. [9]. It is the fraction of transcription factors or genes that are regulated by multiple transcription factors. We adjusted the definition in this paper to investigate the collaboration structures inside a topology, i.e., the degree of collaboration is the fraction of nodes that are regulated by multiple nodes. The degree of collaboration does not depend on the numbers of nodes and links. Bhardwaj et al. [9] introduced two types of degrees of collaboration. The first was the degree of collaboration in each layer D^{L}_{collab} and the second was the degree of collaboration between layers $D^{L_1,L_2}_{betw-level-collab}$.

3.5.1 Degree of Collaboration in Each Layer

The degree of collaboration in each layer D^{L}_{collab} represents the average of D^{i}_{collab} for all nodes i at the L-level, where D^{i}_{collab} is the number of nodes that are co-regulated by node i and another node (A, for instance) divided by the nodes that are regulated by node i. The formal definition of D^{i}_{collab} and D^{L}_{collab} is:

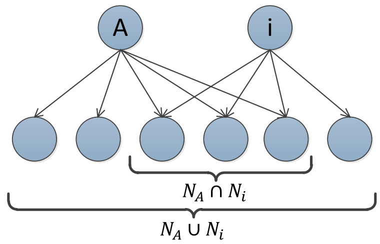

Fig. 4 Collaboration structures between nodes i and A: $|N_A \cap N_i|$ is number of nodes regulated by nodes A and i. $|N_A \cup N_i|$ is number of nodes regulated by node A or node i.

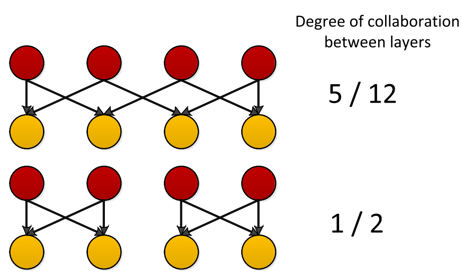

Fig. 5 Illustrative example of how the degree of collaboration between layers defined in [9] differs even when it has the same number of collaboration structure. Both of two topologies (upper and bottom) has the same numbers of nodes/links, and four collaboration structures. In the upper topology, the degree of collaboration between layers is 5/12, while it is 1/2 in the bottom topology. The difference is caused by the term $N_A \cup N_B$ in Eq. (3). We therefore introduce Eq. (4) such that the degree of collaboration between layers is not affected by the term $N_A \cup N_B$.

$$D^{i}_{collab} = \frac{\sum_{A \in N} |N_i \cap N_A|}{|N_i|}, \quad (1)$$

$$D^{L}_{collab} = \langle D^{i}_{collab} \rangle_i \quad \forall i \in L, \quad (2)$$

where N is a set of nodes in the network, and N_i is a set of nodes that are regulated by node i. Then, $|N_i \cap N_A|$ represents the number of nodes that are regulated by both nodes i and A shown in Fig. 4. $\langle\ \rangle$ represents the arithmetic average.

3.5.2 Degree of Collaboration between Layers

The degree of collaboration between layers $D^{L_1,L_2}_{betw-level-collab}$ indicates the fraction of nodes that are co-regulated by the node at the L_1-level and the node at the L_2-level, and is defined by:

$$D^{L_1,L_2}_{betw-level-collab} = \frac{\sum_{A \in L_1} \sum_{B \in L_2} \frac{|N_A \cap N_B|}{|N_A \cup N_B|}}{|L_1| \cdot |L_2|}, \quad (3)$$

where $|N_A \cup N_B|$ is the number of nodes regulated either by node A or by node B (see Fig. 4 for illustrative example). $|L|$ is the number of nodes including in L-level. However, the degree of collaboration between layers in Ref. [9] is affected by structures other than the collaboration structure, which we illustrate in Fig. 5. Both of topologies (upper and bottom) have the same number of nodes/links and the same number of collaboration structures, but have one difference: In the upper graph of Fig. 5, each two nodes co-regulate one node, whereas specific two nodes co-regulate two nodes in the bottom graph. In this case, the original definition (Eq. (3)) differs for two topologies. We therefore modified the definition of the degree of collaboration between layers and introduce Eq. (4) such that the number of collaboration structures is directly counted in order to compare several router-level topologies that have different numbers of nodes/links.

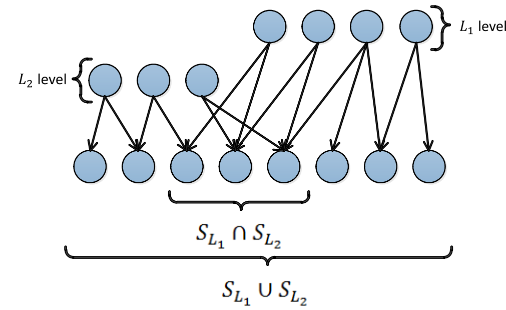

Fig. 6 Modification of definition of degree of collaboration between layers. Here, degree of collaboration between layers is $\frac{3}{8}$.

$$D^{L_1,L_2}_{collab-betw} = \frac{|S_{L_1} \cap S_{L_2}|}{|S_{L_1} \cup S_{L_2}|}, \quad (4)$$

Figure 6 outlines $S_{L_1} \cap S_{L_2}$ and $S_{L_1} \cup S_{L_2}$. S_{L_1} is the number of nodes regulated by nodes in L_1 level. The S_{L_1} is the number of nodes regulated by nodes at the L_1 level. The $|S_{L_1} \cap S_{L_2}|$ is the number of nodes regulated by both a node included in the L_1-level and another node included in the L_2 level. The $|S_{L_1} \cup S_{L_2}|$ is the number of nodes regulated by nodes included in the L_1-level or nodes included in the L_2-level.

To compare several ISP topologies that have different numbers of nodes/links, we modified the definition of the degree of collaboration between layers to represent the number of collaboration structures:

Definition 4 does not depend on the numbers of nodes/links. To compare several ISP router-level topologies that have different numbers of nodes/links, we calculate:

4. Collaboration Structures and Reliability of Router-Level Topologies

We first evaluate the collaboration structures in eight router-level topologies of AT&T, Sprint, Ebone, Exodus, Level3, Telstra, Tiscal, and Verio [12]. For purposes of comparison, we compare the results obtained from the router-level topologies and the five transcriptional regulatory networks of *E. coli*, human, mouse, rat, and yeast. We calculate the hierarchy for each topology, and then obtain the degree of collaboration in each layer and the degree of collaboration between layers. Note that we do not calculate the degree of collaboration related to the bottom-level layer since nodes at the bottom level do not regulate other nodes according to our definition of hierarchy.

We have presented the degree of collaboration in Figs. 7 and 8. From the results of router-level topologies

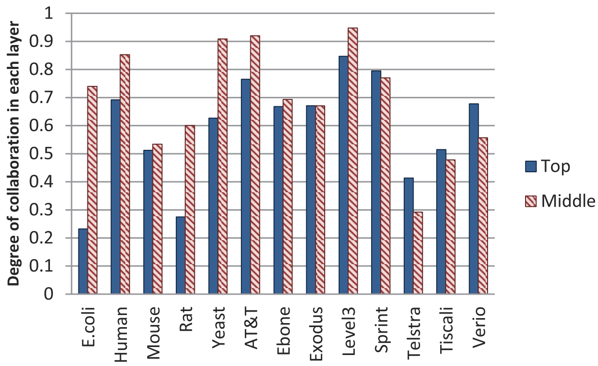

Fig. 7 Degree of collaboration in each layer.

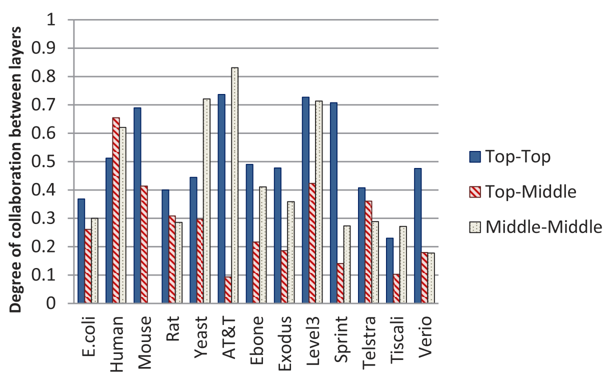

Fig. 8 Degree of collaboration between layers.

in Fig. 7, we can observe that the difference between the degree of collaboration at the top level and the degree of collaboration at the middle level is less than 0.1. In contrast, the difference in transcriptional regulatory networks is generally large. More distinctive characteristics of router-level topologies can be seen from Fig. 8. The collaboration structures between top-level nodes and middle-level nodes are marginal in router-level topologies, whereas these are not in transcriptional regulatory networks. One possible reason for such marginal collaboration structures is the functionality of middle-level nodes in router-level topologies. That is, traffic is first aggregated at middle-level nodes and then forwarded to top-level nodes. Thus, no consideration is given to load-balancing between top-level nodes and middle-level nodes. Although degree of collaboration between top-level and middle-level nodes is comparatively high in Telstra, reliability of it is worst in Fig. 1. The reason is that the number of top-level nodes in Telstra is much fewer than that in other topologies, and there is less degree of collaboration in top and middle layers. With this case and only at the Telstra, the reliability is low because the primal bottleneck (in terms of reliability) is the connectivity between top-level nodes.

Looking at Fig. 7, we observe that most of router-level topologies have high degree of collaboration in each layer. However, the results of Fig. 1 indicate that the reachability from top-level nodes is not high. The reason of decreasing reliability is lacks of collaboration structures between layers. Therefore, it is expected that increasing the collaboration structure between top-level and middle-level nodes improves the reliability. Again referring to Fig. 1, note that these organisms are very reliable. That is, more reliable networks are expected to be constructed by incorporating such collaboration structures. In the next section, we will discuss the effect of collaboration structures on reliability in detail.

5. Effects of Collaboration Structures on Reliability

The previous section explained that the human, mouse, and yeast transcriptional regulatory networks were the most reliable of the organisms we investigated, and we found that these organisms exhibited higher degrees of collaboration between top-level and middle-level nodes, while the router-level topologies exhibited lower degrees of collaboration between them.

This section describes our investigations into what effects collaboration structures have on reliability. More specifically, we increase the collaboration structures between top-level and middle-level nodes by rewiring links in the router-level topologies, and evaluated the differences in reliability before and after the links were rewired. Note that an actual ISP network may increment links or their capacity rather than rewiring them. However, we still consider rewiring links because our prime concern here is whether increasing the number of collaboration structures will improve reliability or not.

5.1 Rewiring to Increase Number of Collaboration Structures

Here, we explain how we rewired links to increase the collaboration structures between top-level and middle-level nodes. The operation consisted of the four steps described below. Each step in rewiring has been outlined in Fig. 9.

Step 1 Find node X regulated by three or more nodes on the same level. If several nodes are found, a node is randomly selected.
Step 2 Randomly select node Y from several nodes that regulate node X and that are at the same level.
Step 3 When node Y is a middle-level node, find node Z that is only co-regulated by top-level nodes. Otherwise, i.e., when node Y is a top-level node, find node Z that is only regulated by a middle-level node. If there are several candidates for node Z, randomly select one of them as node Z.
Step 4 Rewire a link between nodes Y and X; remove the link from node Y to node X, and wire a link from nodes Y and Z.

Note that if node X in Step 1 is selected from nodes only regulated by two nodes, rewiring the link leads to decreased collaboration in the layer (middle-level layer in Fig. 9) that node Y belongs to.

This rewiring is continued until either of the following termination conditions is satisfied.

Condition A When there is no candidate for node X.

Condition B When there are some candidates for node X, but there are no candidates for node Z

Condition C When all nodes are connected to top-level nodes and middle-level nodes, i.e., rewiring is not necessary.

The degree of collaboration between layers after rewiring is summarized in Fig. 10, and it shows that this operation certainly increases the numbers of collaboration structures between top-level and middle-level nodes. Table 3 summarizes the number of rewirings carried out until the algorithm reaches either of the termination conditions. As Table 3 indicates, the number of rewirings until termination conditions are reached differs for the topologies. The reason for this is not only the size of topologies but also the number of candidates for nodes X and Z in Fig. 9. That is because the number of rewirings depends on the number of candidates for nodes X and Z. The types of termination conditions for each topology are also listed in Table 3. The type of termination condition in most router-level topologies, except for Level3, Sprint, and Verio, is condition A, i.e., there are a few candidates for node X in these topologies. For Level3, all the middle-level and bottom-level nodes are connected to top-level and middle-level nodes after rewiring. Since most nodes are connected to higher level nodes before rewiring for Sprint and Verio, there are more candidates for node X and less for node Z.

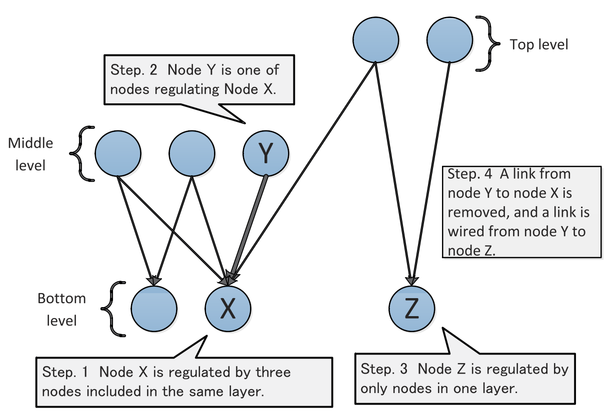

Fig. 9 Steps in link rewiring. There is also a case where node X is connected with three or more top-level nodes, node Y is top level, and node Z is only connected with middle-level nodes.

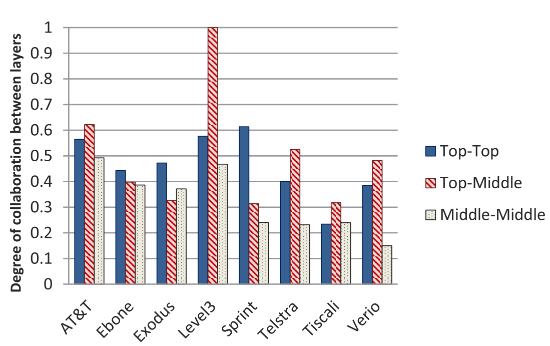

Fig. 10 Degree of collaboration between layers after rewiring.

5.2 Reliability of Topologies after Links are Rewired

Last, we investigate the reliability of topologies after links were rewired, which increased the degree of collaboration between top-level and middle-level nodes. Unlike Fig. 1, which shows the connectivity of directed networks after node failures, we investigate connectivity after random node failures by using the undirected links instead of directed links, and evaluate the difference between the original router-level topologies and topologies after links were rewired. We particularly use the *cover ratio* as the measure of reliability, which is defined as $\frac{S_i}{N}$. The S_i is the number of nodes in the largest connected component after failure in the i-th node, and N is the number of nodes in the original topology. That is, $\frac{S_i}{N}$. The S_i means the ratio of remaining nodes to the number of nodes in the original topology when i nodes have failed. In Sect. 2.2, we used the reachable node ratio for investigating the reliability on a directed network because the transcriptional regulatory networks are directed. However, since router-level topologies are undirected networks, our concern here is the connectivity between nodes. Thus, we use the cover ratio that is defined on undirected networks here.

Figure 11 plots the cover ratios for each topology after the links were rewired. We randomly rewired the links for each router-level topology until the algorithm reached terminal conditions. We obtained three topologies for each router-level topology by applying the rewiring algorithm, and examined 300 trials of random node failures for each of the topologies we obtained. The average of the cover ratios is plotted in the figure, where *Upper bound* represents the maximum cover ratio.

We can see that the cover ratios improve for most router-level topologies, except for Sprint, Exodus, and Level3, which demonstrate little improvement. However, there is no topology where the cover ratio decreases.

We can see that the cover ratios improve in all the router-level topologies. However, the improvements in the cover ratios for Level3, Sprint, and Exodus are marginal. The reasons for this are as follows. The original Level3 topology has numerous links and already has a high cover ratio. That is, it offers little room for improvement. The marginal improvements in the Sprint and Exodus topologies

Table 3 Number of rewirings until termination condition is reached and reached termination conditions for each ISP topology.

Topology	AT&T	Ebone	Exodus	Level3	Sprint	Telstra	Tiscali	Verio
Number of rewirings	222	15	15	154	59	48	36	170
Types of termination conditions	A	A	A	C	B	A	A	B

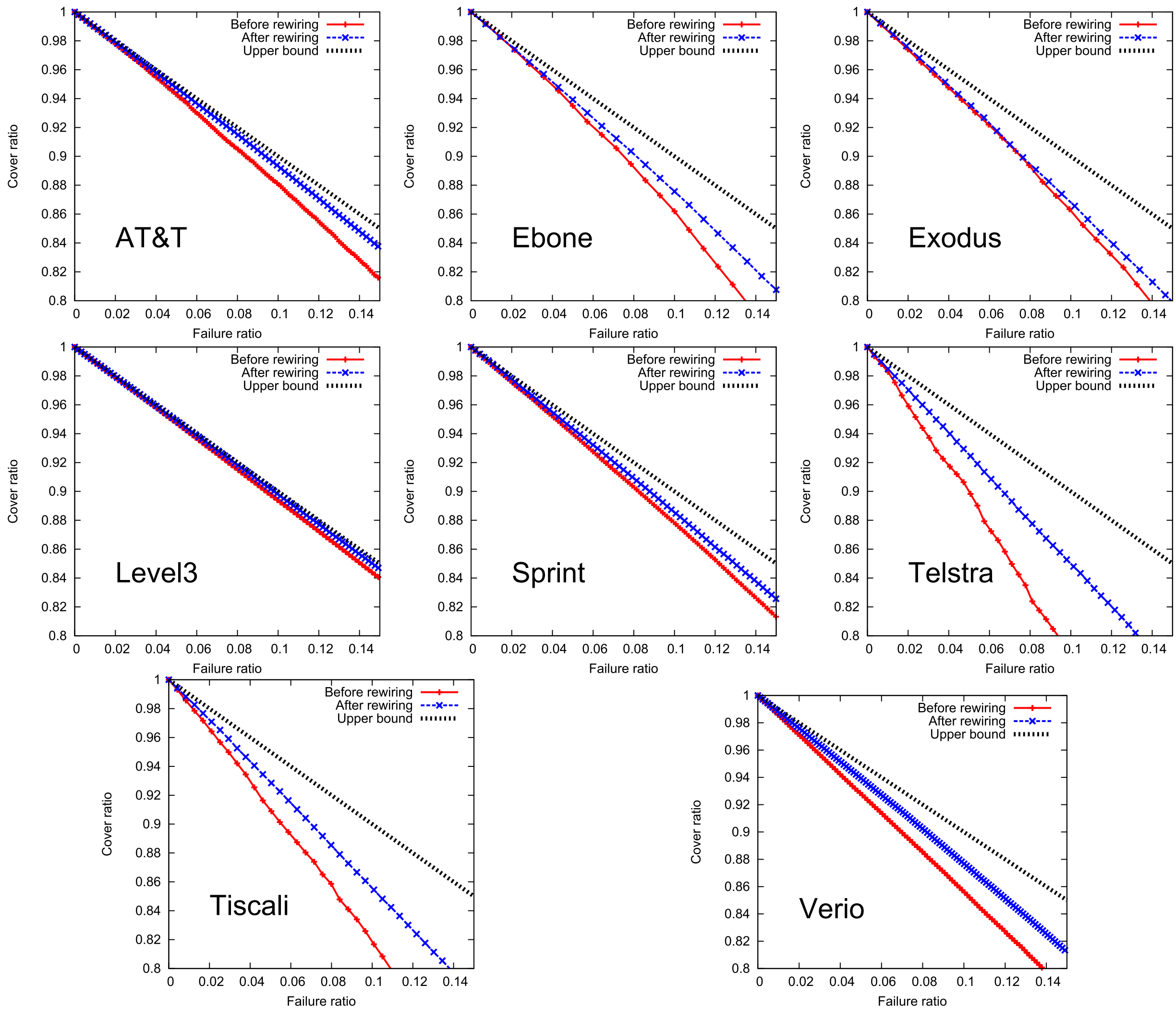

Fig. 11 Difference in reliability between topologies before and after rewiring.

are caused by the poor opportunities for rewiring. A few nodes in the Sprint topology are only connected to middle-level nodes. Hence, the Sprint topology has few candidates for node Z in Fig. 9. There are few candidates for node X in Fig. 9 in the Exodus topology because most nodes do not have three or more links connected to top-level nodes and they do not have three or more links connected to middle-level nodes. Note that, the cover ratio in the Ebone topology improves more than that in the Exodus topology even though the number of rewirings is the same for both topologies. This is because the degree of collaboration in Ebone increases more through rewiring than that in Exodus. As summarized in Table 1, the number of nodes and links in Ebone is less than that in Exodus, but the number of rewirings is the same as that in Ebone and Exodus. Thus, the degree of collaboration in Ebone increases more. Because Ebone obtains more collaboration structures under the given number of nodes and links compared with Exodus through rewiring, the cover ratio in Ebone is improved more than that in Exodus.

The results in this section indicate that the collaboration structures of topologies characterize reliability, and reliability improves to some extent by increasing the number of collaboration structures.

6. Conclusion

We investigated collaboration structure in router-level topologies, and found that there were fewer collaboration structures between top-level and middle-level nodes in router-level topologies than those in transcriptional regulated networks. Because of this, the connectivity of router-level topology easily deteriorated when node failures occurred. We demonstrated that the reliability of several topologies improved when the collaboration structures between top-level nodes and middle-level nodes increased to find a possible evolutionary path to improve the reliability of router-level topologies. However, the improvements to reliability were limited in Level3, Sprint, and Exodus topologies. These topologies were extremely reliable before rewiring. In other words, if original router-level topologies are not reliable, this is more likely to improve reliability. We investigated collaborative structure in router-level topologies, and found that there were fewer collaborative structures between top-level and middle-level nodes in router-level topologies than those in transcriptional regulated networks as we have discussed in Ref. [15].

Our future work is to establish network designs based on collaboration structures for large-scale and reliable router-level topologies. We investigated the relationship between collaboration structures and the reliability of networks by rewiring links in this research. However, link rewiring may be impractical for network design because ISPs do not need to remove old links. Incorporating the property of collaboration structure to evolving strategies, such as [16], [17] may be important, but it is left for future investigations.

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