Improving Reliability of Inter-connected Networks through Connecting Structure

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***Abstract*—The Internet plays an important role in our life as social infrastructure, and the importance of reliability is widely recognized in the Internet. There are many studies on network design with high reliability but most of them intend for constructing a single network that a network operator governs. However, the Internet consists of many of small networks, which are mutually connected. Therefore, it is important to enhance reliability of inter-connected network consisted from two or more networks rather than focusing only on the reliability inside the single network. In this paper, we show how we should connect two networks for achieving high reliability of inter-connected network. We evaluate the reliability with various kinds of connecting structures. Evaluation results show that high reliability is achieved by a multiscale structure where links for inter-connection are prepared for connecting nodes belonging to different hierarchical level in the network.**

Keywords*—*Power-law Networks; BA Model; Reliability; Connecting Structure; Multiscale Structure.

I. Introduction

The number of users connected to the Internet is increasing through mobile terminals and various services such as social networking service are deployed. The Internet plays an important role in our life as social infrastructure, and therefore reliability is one of the important characteristics for the Internet.

Internet Service Providers (ISPs) construct their own networks to accommodate the traffic of customers with a minimum of equipment costs while keeping the reliability against failures of equipment [1]. A key functionality to keep the reliability is the restoration, i.e., re-route packets when failures occur. Network operator of ISP envisions kind of failures and then designs physical topology and capacity of links so that the network works under the envisioned failures. However, when more significant failures than initially envisioned, the network becomes out of control, that is, it may work or may not work. The network operator faces on the difficulty in deciding the scale of failures of undertakings.

In previous studies, a single node failure and/or single link failure were supposed as the failure of equipment [2], [3]. The fundamental approach of these studies is to enumerate all of failure patterns and then prepares physical links or determines the capacity of links to accommodate traffic demand for all of failure patterns. However, it is easily imagined that such the approach encounters the difficulty in designing networks when the size of simultaneous failures envisioned increases. The reliability against multiple node/link failures is investigated in [4], [5]. They focus on the statistical characteristics of topology and investigate the relation between the characteristics and the reliability under the multiple failures. Results show that the power-law network where the probability of existence of nodes having k links is proportional to $k^{-\gamma}$ (γ is constant) loses its connectivity easily when nodes with high degree are failed, but the power-law network is reliable against random node failures.

Above studies intend for enhancing reliability of an ISP network that the network operator governs. However, the reliability of the Internet is achieved not only by the enhancing reliability of ISP networks but also by enhancing reliability of inter-connected network, where two or more ISP networks are mutually connected, since the Internet consists of many of ISPs which are mutually connected. In this paper, we investigate the reliability of inter-connected network that consists from two networks and their connecting links. Hereafter, we will call the inter-connected network as global network, and call its consisting networks as local networks. Our concern is how we should connect a limited number of inter-connected links between local networks to make the global network to be reliable against multiple failures. Note that we evaluate connecting structure between local networks rather than the topological structure of local network itself, since the reliability of local network has been investigated in the above studies.

Recently, the reliability of electronic network that consists from power-grid network and its control network is discussed [6]–[8]. Since the control network requires the power from the power-grid network, the authors investigate that how to inter-connect two networks such that reliability against cascade failures is maximized. The cascade failure is successive failures caused by a cascade of power-outage which is triggered by an initial failure point. They pointed out that the global network is reliable against cascade failures when two local networks are connected with links through "similar" nodes. That is, inter-connected links should be prepared between nodes with similar degree or similar clustering coefficient. Unlike the electronic network where nodes of control network must be connected with the power-grid network, communication network does not require full connectivity between two networks. Rather, it is important for communication networks to reduce the number of inter-connected links to keep the reliability to some extent. Note again that our concern of this paper is how we should connect a limited number of inter-connected links between local networks, which is particular to communication networks.

This paper is organized as follows. We introduce related work of this paper in Section II. Section III shows the topology model that we use for the evaluations. In Section IV, we evaluate reliability with various classes of connecting structure against node failures. Finally, Section V concludes this paper and mentions the future work.

II. Related Work

Dodds et. al. showed a network construction algorithm that constructs five classes of networks and compared their robustness [4]. The algorithm starts from a hierarchical tree topology with branching ratio b and L levels of branching. Then, the algorithm adds m links chosen stochastically with a probability. The probability that there exists a link between two nodes, say i and j, is denoted as $P(i,j)$ and is determined by the depth D_{ij} of their nearest common ancestor a_{ij}. The probability is also determined by node's own depths d_i and d_j (Fig. 1). Formally the probability $P(i,j)$ is defined as,

$$P(i,j) \quad \propto \quad e^{-D_{ij}/\lambda} e^{-x_{ij}/\zeta}, \qquad (1)$$

where λ and ζ are tunable parameters. x_{ij} represents the distance between two nodes i and j and is set to $(d_i^2+d_j^2-2)^{1/2}$, which represents relative distance in the hierarchy [4]. By changing the values of λ and ζ, this algorithm generates topologies with various topological structures. The authors categorized generated topologies into the following five classes.

- Random (R) by setting $(\lambda, \zeta) \to (\infty, \infty)$: links are added randomly.
- Random interdivisional (RID) by setting $(\lambda, \zeta) \to (0, \infty)$: more links are added for smaller value of D_{ij}, but do not take care of x_{ij}. That is, the link between nodes that have large distance.
- Local Team (LT) by setting $(\lambda, \zeta) \to (\infty, 0)$: links tend to be added between nodes that have short distance, regardless of their layer in hierarchy.
- Core-periphery (CP) by setting $(\lambda, \zeta) \to (0, 0)$: links tend to be added between nodes located at higher-level in hierarchy, and between nodes that have short distance. The resulting topology exhibits densely connected "core" and sparsely connected "edge" network.
- Multiscale (MS) with intermediate values of λ ($0 < \lambda <$1) and ζ ($0 < \zeta <$1). The resulting topology has connectivity dominated by the range from a small x_{ij} to a large x_{ij}. The resulting topology has a property that the link density decreases as the hierarchical level decreases.

The authors evaluated two kinds of robustness. One is congestion robustness and the other is connectivity robustness. Congestion robustness is measured by the maximum congestion that imposes a load of packet processing at node. Connectivity robustness represents the size of the largest connected component remaining after failures. Their evaluation reveals that the multiscale structure improves both the congestion robustness and connectivity robustness.

III. Connecting structure for inter-connected network

In this section, we present a model of connecting structure between two local networks inspired by the network construction algorithm explained in the previous section. There are two local networks: local network A and local network B. The global network (inter-connected network) is formed by connecting links between A and B. Depending on a strategy where to connect, various connecting structure can be arranged.

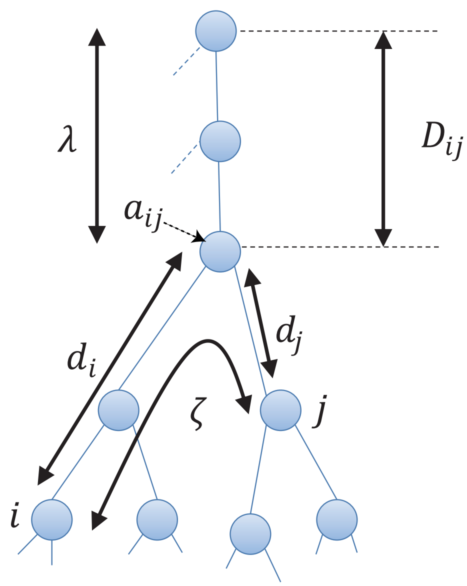


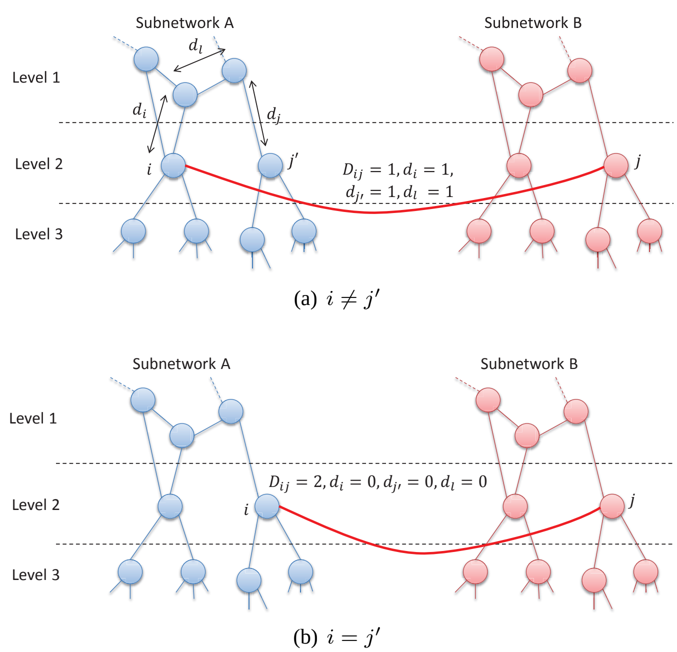
Fig. 1. Illustrative example of the network construction algorithm ($D_{ij} = 2$, $d_i = 2$, and $d_j = 1$)

For developing the model, we assume that two local networks are identical. Note that such the assumption does not reflect the actual network. However, we use the assumption in this paper since our main concern is to reveal fundamental reliability of inter-connected network and investigate differences of the reliability on various connecting structures. Actually, the reliability may be different dependent on things of each local network. We will consider networks having different topology as a future work. We also assume that the local network has a hierarchy structure and has a level of hierarchy.

Let us consider that the probability $P(i,j)$ which represent the probability of link existence between node i from local network A and node j from local network B. Then, we calculate connection probability $P(i,j)$ of all nodes pairs (i,j), which is defined as,

$$P(i,j) \propto e^{-D_{ij}/\lambda} e^{-x_{ij}/\zeta}. \qquad (1)$$

Note that this equation is the same to the equation in [4]. However, we change the definition of each notation to apply our problem that connects two local networks. First, we redefine the distance x_{ij} by using three values d_i, d_j, and d_l. Hereafter, we use a node j' of local network A instead of a node j of local network B. Node j' of local network A corresponds to the node j of local network B. Note again that we assume that local networks A and B are identical to reveal the reliability of global network. In our model, d_i is defined as the number of upstream hops in the shortest path from source node i to a common ancestor $a_{ij'}$. Similarly, we define d_j as the number of downstream hops from a common ancestor $a'_{ij'}$ to node j. d_l represents horizontal distance in the hierarchical local network and is set to the shortest hop length between i and j excluding d_i and d_j. In this paper, we introduce a concept of horizontal distance to consider a non-tree-based topology as the local network. Dodds et al. [4] consider the tree-based topologies for network construction and the non-tree-based topology is not treated. Illustrative example of d_i,



(a) $i \neq j'$

(b) $i = j'$

Fig. 2. Definition of x_{ij} and D_{ij} used for connecting two local networks

TABLE I. VALUES OF (λ, ζ)

notation of connecting structure	(λ, ζ)
Random (R)	(∞, ∞)
Local Team (LT)	$(\infty, 0.05)$
Random Interdivisional (RID)	$(0.05, \infty)$
Core-periphery (CP)	(0.05, 0.05)
Multiscale (MS)	(0.5, 0.5)

d_j, and d_l, is shown in Fig. 2. Then, the distance x_{ij} is re-defined as $(d_i^2 + d_j^2 + d_l^2)^{1/2}$.

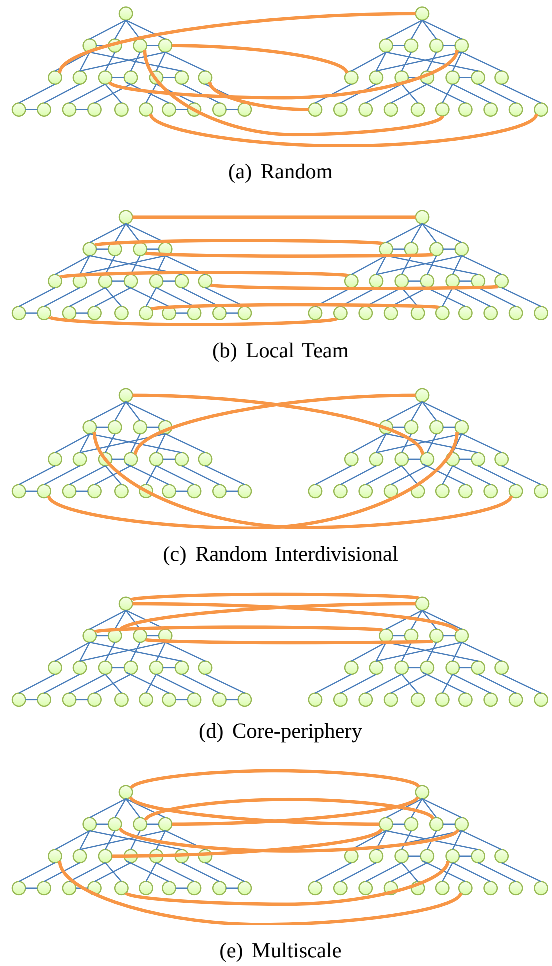
After calculating connection probability, we connect two nodes belonged to different local networks. We select connected nodes pair i and j according to $P(i, j)$. We then connect between node i in local network A and node j in local network B. We repeat adding links between two local networks until the number of inter-connected links reaches m.

By changing the parameters λ and ζ, we generate some classes of inter-connected topology. We use the same definition of classes in the way of [4] (shown in Table I and Fig. 3). However, Multiscale structure is defined as the middle parameters of other four structures, so we cannot set the unique value for Multiscale structure. Therefore, we evaluate some parameters other than $(\lambda, \zeta) = (0.5, 0.5)$. We set the number of inter-connected links to 50, 100, and 200.

IV. Reliability Evaluation of Inter-connected Network

A. Local Network

We prepare a local network based on BA model [9]. BA model is a well-known generation model for topology whose degree distribution follows a power-law. The BA model incrementally adds a new node, and the new node connects with existing nodes by a preferential manner, i.e., new nodes tends to connect higher degree node. The detailed of algorithm to generate the BA topology is as follows:



(a) Random

(b) Local Team

(c) Random Interdivisional

(d) Core-periphery

(e) Multiscale

Fig. 3. Five classes of connecting structure obtained by changing parameters, λ or ζ.

1) Prepare a complete graph with m_0 nodes
2) Repeat following processes until the number of nodes equal to n
   a) set a new node
   b) select m nodes with the probability $k_i / \sum_j k_j$ (k_i is the degree of node i) and connect between selected nodes and a new node.

In this paper, we consider four patterns of local network by changing values of (n, m) to (500, 2), (500, 3), (1000, 2), (1000, 3). m_0 is set to 3 for all patterns. Hierarchical level of BA topology is defined by the hop count from the node with largest degree in the local network.

B. Performance Metrics

We evaluate the average hop length and the connectivity when multiple failures occur. Hereafter, N denotes the number of nodes and B denotes the largest connected component after the failures occur.

- Average hop length H
  H denotes the average hop length for all pairs of

nodes, which is defined as

$$H = \frac{\sum_{i \in B}^{N} \sum_{j \neq i, j \in B}^{N} d_{ij}}{|B|(|B|-1)}, \quad (2)$$

where d_{ij} is the shortest hop length from node i to node j calculated by Dijkstra's shortest path algorithm.

- Connectivity C
  C denotes the ratio of the number of nodes in B to a set of all survived nodes, which is defined as

$$C = \frac{|B|}{N - |r|}, \quad (3)$$

where r is a set of failed nodes. $|B|$ and $|r|$ means the number of elements in each set.

C. Reliability against node Failures

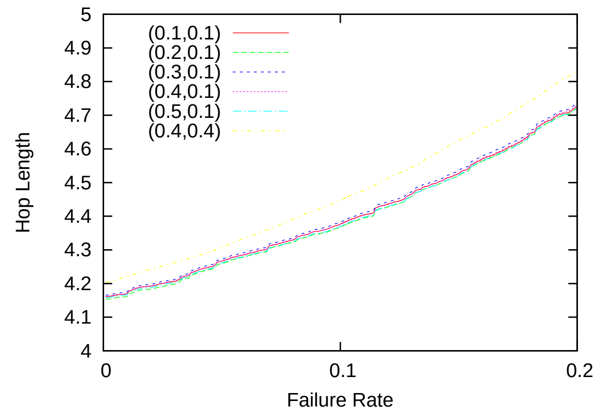
In this section, we consider the scenario that a node failure occurs at random one by one.

1) Parameter settings for Multiscale Structure: As we discussed in Section II, Multiscale structure is intermediate of other four structures (Random, Local Team, Random Interdivisional, Core-periphery). Since the parameters λ and ζ takes various values, we first investigate the best values of the parameters for multiple node failures. In [4], setting λ to 0.5 and $zeta$ to 0.5 exhibits best parameter setting for improving robustness for constructing a local network. A question of this paper is whether setting λ to 0.5 and ζ to 0.5 is best or not.

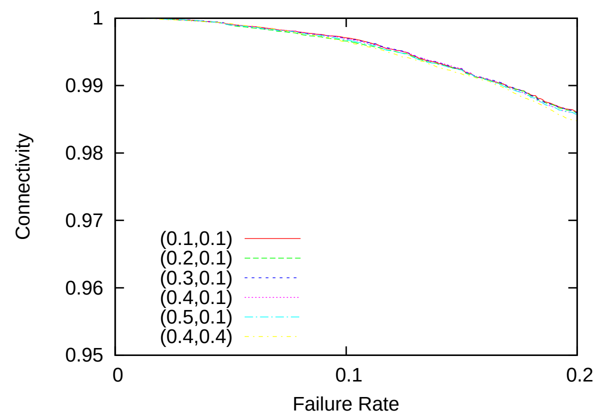
In [4], congestion robustness is improved when the Multiscale structure close to the Core-periphery structure. We therefore investigate the parameter set which is close to Core-periphery structure. More specifically, we evaluate reliability by changing λ and ζ from 0.1 to 0.5 by 0.1, respectively. We calculate average of C and H for 100 patterns of local networks having 500 nodes with average degree 2. The number of inter-connected links is set to 200.

For obtaining best parameter settings, we change ζ from 0 to 0.5 while λ is fixed. When λ is set to 0.1, 0.1 for $zeta$ exhibits best reliability in terms of connectivity and average hop count. We next set λ to 0.2, but again 0.1 for ζ exhibits best reliability. Finally, we obtained that setting ζ to 0.1 is best for λ 0.1, 0.2, 0.3, 0.5, except when λ is set to 0.4; setting ζ to 0.4 exhibits best reliability. Figure 4 shows connectivity C and average hop-count H for different value of λ. For each value of λ, ζ is chosen such that the reliability is maximized. Comparing results with various λ, we observe that the average hop length is minimized when ($of\lambda$, ζ) is (0.1, 0.1), but this is so close to Core-periphery structure that we do not select the parameters as Multiscale structure. Though we cannot see notable differences on the average hop length and the connectivity, we select is (0.3, 0.1) as the parameter (λ, ζ) since the connectivity C is slightly higher than other parameters.

The reason for showing high reliability is that MS (0.3, 0.1) has more inter-connected links that connect nodes at 2nd layer than MS (0.5, 0.5). To clarify this, we show the number of nodes in each layer in Table II, and the number of inter-connected links dependent on layers in the hierarchy in Table III (MS (0.5, 0.5)) and Table IV (MS (0.3, 0.1)). When (λ, ζ) is set to (0.3, 0.1), the number of inter-connected links related to 2nd layer is 72, which is larger than the case of MS (0.5, 0.5). In the case of BA topology, nodes at 2nd layer tend to connect with node at 1st layer, i.e., largest degree node. The average hop length is therefore decreased for MS (0.3, 0.1).



(a) average hop length



(b) connectivity

Fig. 4. reliability of topology formed by (λ, ζ) = (*, 0.1), (0.4, 0.4)

From these observations, we investigate MS (0.3,0.1), which we set λ to 0.3 and ζ to 0.1, as well as MS (0.5, 0.5) for Multiscale structure.

TABLE II. THE NUMBER OF NODES IN EACH LAYER: 500 NODES, AVERAGE DEGREE 2 FOR LOCAL NETWORK

Hierarchical level	number of nodes
1	1
2	61
3	169
4	228
5	41

TABLE III. LAYERS WITH INTER-CONNECTED LINKS OF NETWORKS BY ($\lambda = 0.5, \zeta = 0.5$)

Hierarchical level	1	2	3	4	5
1	0	0	0	2	0
2	0	6	8	12	3
3	0	9	23	28	8
4	1	10	27	45	10
5	0	1	2	3	2

TABLE IV. LAYERS WITH INTER-CONNECTED LINKS OF NETWORKS BY ($\lambda = 0.3$, $\zeta = 0.1$)

Hierarchical level	1	2	3	4	5
1	0	0	0	0	0
2	1	10	21	12	1
3	0	17	29	20	6
4	0	9	28	33	2
5	0	1	5	4	1

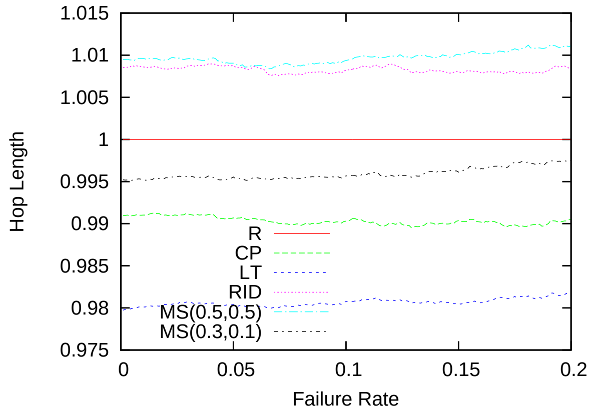


Fig. 5. Average hop length for multiple failures: 500 nodes, average degree 2 for local networks; 50 inter-connected links.

2) Evaluation on Connecting Structure: We evaluate reliability of networks with MS (0.3, 0.1) in addition to five classes of connecting structures shown in Table I. We show the average hop length in Fig. 5 with 500 nodes and average degree 2 for local networks. In this figure, X-axis shows the ratio of node failures and Y-axis shows average hop length normalized by the result of Random structure. We observe that the structures with dense links in upper layers, such as Core-periphery structure or Local Team structure, could make the average hop length to be low. However, when we change the number of nodes or links in local networks, average hop length H of Core-periphery structure and Local Team structure get worse, and sometimes close to that of Random structure. MS (0.3, 0.1) can keep the average hop length low regardless of the number of nodes or links used for local networks.

Next, we show the connectivity C in Fig. 6. In this figure, X-axis shows the ratio of node failures and Y-axis shows connectivity C normalized by the result of Random structure. We can see that MS (0.3, 0.1) or MS (0.5, 0.5) show higher connectivity than that of other structures.

We also show worst case of connectivity C and average hop length H in Figs. 7 and 8. The definition of X-axis and Y-axis is the same to the definition of Fig. 6. As shown in Fig. 7, we cannot observe any remarkable differences among results of each connecting structure. This is because we use BA topology as local networks. BA topology has degree distribution obeying a power-law and the topology already has a robustness against random node failures. However, Fig. 8 shows that MS (0.3,0.1) can take higher connectivity C than other structures against failure rate. These results show that MS (0.3,0.1) showed low average hop length and high connectivity when multiple node failures occur.

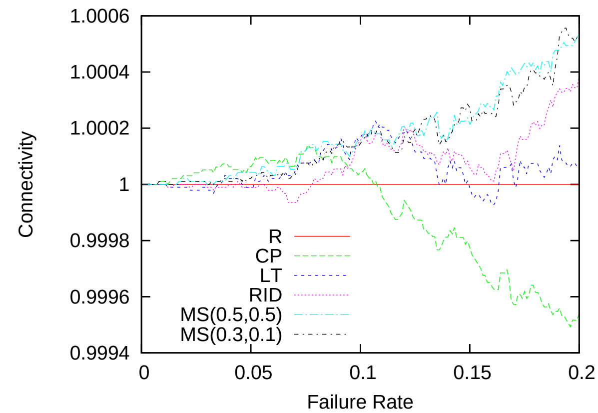


Fig. 6. Connectivity for multiple failures: 500 nodes, average degree 2 for local networks; 50 inter-connected links.

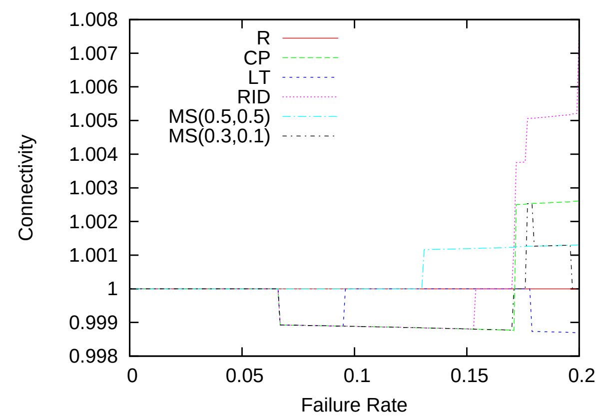


Fig. 7. Worst case of connectivity C for multiple node failures: 500 nodes, average degree 2 for local network; 50 inter-connected links

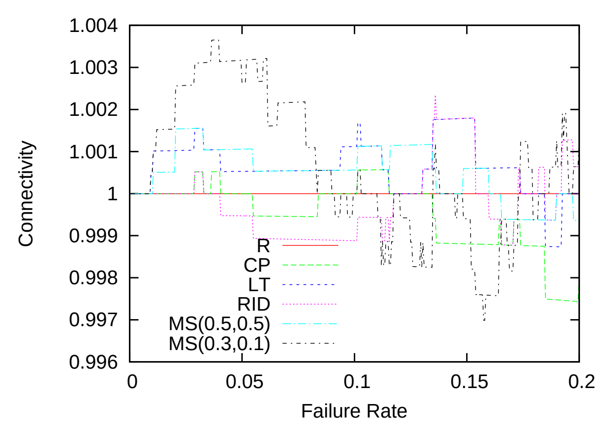


Fig. 8. Worst case of connectivity C for multiple node failures: 1000 nodes, average degree 2 for local network; 200 inter-connected links

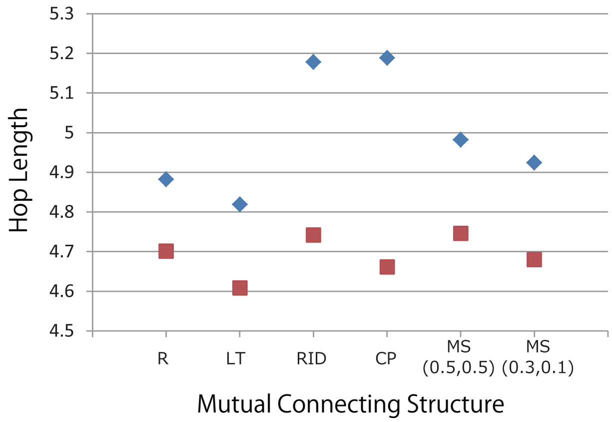


Fig. 9. The average hop length for disaster failure: 500 nodes, average degree 2; 50 inter-connected links). ■ shows the average and ◆ shows the worst values

D. Reliability for Disaster Failures

In the previous section, we evaluated reliability against multiple node failures where a single node failure successively and randomly occurs one by one. This section evaluates reliability of inter-connected network against disaster failure. As opposed to random node failures examined at previous section, we consider multiple node failures where a selected node and its neighbor nodes fail simultaneously. For evaluating the reliability against disaster failure, we consider failures of largest degree node and its neighbor nodes. This is the worst case scenario for the disaster failure since the scale of disaster is largest. Of course, it is possible to occur multiple disasters at the same time, but the possibility is extremely low, so we do not evaluate multiple disaster scenario.

We examined various local networks by changing the parameter for generating BA topology. Figure 9 is the results when we use 500-node with average degree 2 as the local network. In this figure, X-axis represents classes of connecting structure, and Y-axis represents the average hop length H. Worst and average of H over 100 patterns of local network is presented for each class of connecting structure. The results show that MS (0.3, 0.1) and Local Team structure can keep the average hop length low for both worst and averaged results. It is also revealed that Core-periphery structure and Random Interdivisional structure takes high average hop length at the worst case. We had the same observation for other local networks, so omitted results in this paper. We also omit evaluations of connectivity C since we cannot see remarkable differences among connecting structures.

Based on these results, we conclude that MS (0.3, 0.1) structure shows high reliability for multiple node failures and a disaster failure. That is, high reliability of inter-connected is achieved by connecting nodes belonged to different hierarchical level in local network and by connecting nodes around the core of local network densely.

V. Conclusion and Future Work

In this paper, we revealed how we should connect two local networks for achieving high reliability of inter-connected network. For this purpose, we extend the algorithm in [4] with re-definition of distance x_{ij} between nodes i and j. We then examined various classes of connecting structures between two local networks, and evaluate the connectivity and average hop length after multiple node failures. The results showed that high reliability is achieved by MS (0.3, 0.1), which is the Multiscale structure with λ 0.3 and ζ 0.1. The other structures sometimes take high reliability, but MS (0.3, 0.1) always takes high reliability.

In the future work, we will investigate the reliability of inter-connected network between two ISP topologies other than BA topologies, and extend the definition of the probability $P(i, j)$ to be applied to connect two local network whose topologies are different from each other.

Acknowledgment

This work was supported in part by Grant-in-Aid for Scientific Research (A) 24240010 of the Japan Society for the Promotion of Science (JSPS) in Japan.

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