Characteristic Analysis of Response Threshold Model and its Application for Self-organizing Network Control

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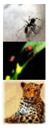
Outline of our presentation

- Background and objective
- Division of labors in social insects
- Hypothetical system as an application
- Mathematical analysis of the system
- Conclusion and future work

Background of our research

- Rapid growth of networks in size, complexity, and dynamics
  - Network control must be robust, adaptive, and scalable
- Biologically inspired self-organizing network control mechanism
  - Many successful attempts published in literatures show its usefulness[6]

Biology
- Foraging in ants
- Synchronization in fireflies
- Pattern formation on surface
- etc.



Apply math models to

Network
- Routing
- Time synchronization
- Topology formation
- etc.

[6] F. Dressler and O. B. Akan, "A survey on bio-inspired networking," Computer Networks, vol. 54, pp. 881–900, Apr. 2010.

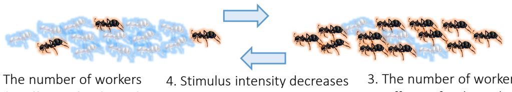
Background and objective of our research

- Biologically inspired self-organizing network control is not necessarily versatile
  - Perturbations cause unintentional results
  - e.g. node failure, message loss and delay
- Deep understanding of mathematical model of biological behavior is needed in regard to fundamental limits and applicability to network control

Take division of labors of social insects as an example of biological behavior
Analyze influence of information loss on recovery phase from individuals failures

Division of labors in a colony of social insects

- Colony of social insects is divided into two groups (workers and non-workers)
- Group size is adjusted to meet the task associated-stimulus intensity



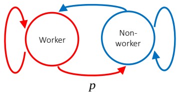
Response threshold model[10]

- Mathematical model of division of labors in social insects
- Dynamics of stimulus intensity $s(k)$ of a task at time k

$$s(k+1) = s(k) + \delta - \frac{\text{the number of workers}}{\text{the number of individuals}}$$

- Stochastic decision on an individual at time k

$1-p$ $\quad \frac{s(k)^2}{s(k)^2+\theta^2} \quad 1-\frac{s(k)^2}{s(k)^2+\theta^2}$



Parameters of response threshold model

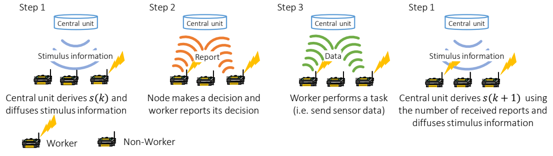
Notation	Range	Description
δ	$0<\delta<1$	Increasing rate of stimulus intensity
θ	$0<\theta$	Hesitation for a non-worker to perform a task
p	$0<p<1$	Probability of quitting a task

[10] E. Bonabeau, A. Sobkowski, G. Theraulaz, J. L. Deneubourg, Adaptive task allocation inspired by a model of division of labor in social insects, in Proceedings of Biocomputing and Emergent Computation, 1997, pp. 36–45.

Hypothetical system as an application for analysis

- Consist of a central unit and nodes (workers/non-workers)
  - Central unit diffuses stimulus information
  - Worker performs a task (e.g. sensing)
  - Non-worker doesn't perform a task
- Periodically repeat three steps

Example: assign sensing task to two sensor nodes



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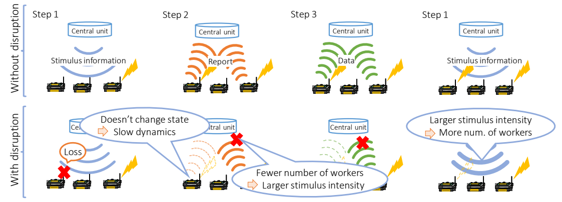
Three occasions for disruption of communication

- Node fails in receiving stimulus information from a central unit
- Central unit fails in receiving a report from a worker
- Central unit fails in receiving a data from a worker



Clarify range of loss ratio for response threshold model to be effective

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Analytical model considering information loss

Notation	Description
δ	Increasing rate of s
M	The number of individuals
D	The number of dead individuals
θ	Hesitation to perform a task

- Loss and reception are considered as Bernoulli trials
- Dynamics can be written using two variables
  - Expected value s of stimulus intensity

Expected number of received reports

$(1-q_w)\cdot n_w$ — n_w: Expected number of workers; $(1-q_w)$: Probability that a central unit receives a report

$$\frac{ds}{dt} = \delta - \frac{(1-q_w)\cdot n_w}{M}$$

  - Expected value n_w of the number of workers

Expected number of workers which received stimulus

$(1-q_s)\cdot n_w$ — n_w: Expected number of workers; $(1-q_s)$: Probability that a central unit receives a report

Expected number of non-workers which received stimulus

$(1-q_s)\cdot(M-D-n_w)$ — $(M-D-n_w)$: Expected number of workers; $(1-q_s)$: Probability that a central unit receives a report

Expected number of workers which quit a task

$(1-q_s)\cdot n_w\cdot p$ — p: Probability of quitting a task

Expected number of non-workers which begin a task

$(1-q_s)\cdot(M-D-n_w)\cdot\frac{s^2}{s^2+\theta^2}$ — $\frac{s^2}{s^2+\theta^2}$: Probability of beginning a task

$$\frac{dn_w}{dt} = -(1-q_s)\cdot n_w\cdot p + (1-q_s)\cdot(M-D-n_w)\cdot\frac{s^2}{s^2+\theta^2}$$

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Parameter area where an equilibrium state exists

Notation	Description
s	Stimulus intensity
δ	Increasing rate of s
q_w	Loss ratio of a report
q_s	Loss ratio of a stimulus
M	The number of individuals
D	The number of dead individuals
n_w	The expected number of workers
θ	Hesitation to perform a task

- Derivation of an equilibrium state $[\bar{s}\ \bar{n}_w]^T$
  - Two equations hold in $[\bar{s}\ \bar{n}_w]^T$

$$\frac{ds}{dt} = \delta - \frac{(1-q_w)\bar{n}_w}{M} = 0 \quad \cdots\text{(A)}$$

$$\frac{dn_w}{dt} = -p(1-q_s)\bar{n}_w + \frac{\bar{s}^2}{\bar{s}^2+\theta^2}(1-q_s)(M-D-\bar{n}_w) = 0 \quad \cdots\text{(B)}$$

  - From Eq. (A) $\bar{n}_w = \delta M/(1-q_w)$

For $\bar{n}_w < M-D$, $(1-D/M)(1-q_w) \geq \delta$ must hold $\cdots$(C)

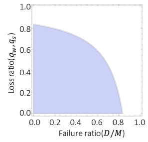
  - From Eq. (B) $\bar{s} = \theta\sqrt{\frac{p\delta}{(1-D/M)(1-q_w)-\delta(1+p)}}$

$(1-D/M)(1-q_w)-\delta(1+p) > 0$ must hold $\cdots$(D)

- Existence condition of $[\bar{s}\ \bar{n}_w]^T$
  - From Eq. (C) and Eq. (D)

$$(1-D/M)(1-q_w)-\delta(1+p) > 0 \quad \cdots\text{(E)}$$

Parameter area where Eq. (E) holds when δ is set at 0.15 and p is set at 0.1.



Equilibrium state exists in wide range of loss and failure ratio

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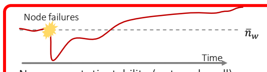
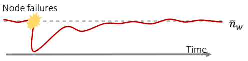
Dynamics toward the equilibrium state



Non-asymptotic stability (not work well)

All real parts of λ aren't negative



Asymptotic stability (work well)

All real parts of λ are negative

$d\vec{x}/dt = A\vec{x}$

Linearly converting

$d\vec{z}/dt = A\vec{z} = \vec{\lambda}\vec{z}$

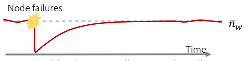
$\lambda_i = \alpha_i + j\beta_i$

$z_i(k) = z_i(0)e^{(\alpha_i+j\beta_i)k} = z(0)e^{\alpha_i k}(\cos\beta_i k + j\sin\beta_i k)$

If all real parts of λ are negative, dynamics of $\vec{x}$ is converges to 0

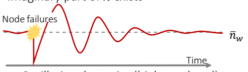
If imaginary part of λ exists, dynamics of $\vec{x}$ ocillates

Imaginary part of λ doesn't exist



Stable dynamics (low overhead)

Imaginary part of λ exists



Oscillating dynamics (high overhead)

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Asymptotic or non-asymptotic stability (1/2)

- Derivation of dynamics $[e_s\ e_w]^T = [s\ n_w]^T - [\bar{s}\ \bar{n}_w]^T$ linearized at $[\bar{s}\ \bar{n}_w]^T$

$$\frac{d}{dt}\begin{bmatrix} e_s \\ e_w \end{bmatrix} = \begin{bmatrix} 0 & 1-q_w \\ \frac{(1-q_s)(1-D/M-\bar{n}_w)2\bar{s}\theta^2}{(\bar{s}^2+\theta^2)^2} & (1-q_s)\left(p+\frac{\bar{s}^2}{\bar{s}^2+\theta^2}\right) \end{bmatrix}\begin{bmatrix} e_s \\ e_w \end{bmatrix}$$

where $\bar{s} = \theta\sqrt{\frac{p\delta}{(1-D/M)(1-q_w)-\delta(1+p)}}$ and $\bar{n}_w = \delta M/(1-q_w)$

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δ	Increasing rate of s
q_w	Loss ratio of a report
q_s	Loss ratio of a stimulus
M	The number of individuals
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n_w	The expected number of workers
θ	Hesitation to perform a task

- Eigenvalues $\lambda_\pm$ of matrix $A = [0\ a; b\ c]^T$ are $\frac{c\pm\sqrt{c^2+4ab}}{2}$

$$c = -(1-q_s)p\left\{\frac{(1-D/M)(1-q_w)}{(1-D/M)(1-q_w)-\delta}\right\}$$

$$ab = -\frac{2\sqrt{p\delta}(1-q_s)\{(1-D/M)(1-q_w)-\delta(1+p)\}^{1.5}}{\theta\{(1-D/M)(1-q_w)-\delta\}}$$

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Asymptotic or non-asymptotic stability (2/2)

- Condition that real parts of all eigenvalues $\lambda_\pm = \frac{c \pm \sqrt{c^2+4ab}}{2}$ are negative
  - Values c and ab are less than zero

$$c = -(1-q_s)p\left\{\frac{(1-D/M)(1-q_w)}{(1-D/M)(1-q_w)-\delta}\right\}$$

$$ab = -\frac{2\sqrt{p\delta}(1-q_s)\{(1-D/M)(1-q_w)-\delta(1+p)\}^{1.5}}{\theta\{(1-D/M)(1-q_w)-\delta\}}$$

Notation	Description
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M	The number of individuals
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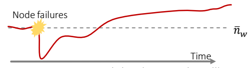
n_w	The expected number of workers
θ	Hesitation to perform a task

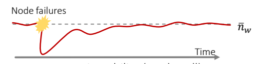
  - From existence condition $(1-D/M)(1-q_w)-\delta(1+p) > 0$ of $[\bar{s}\ \bar{n}_w]^T$, $(1-D/M)(1-q_w)-\delta > 0$ holds

> - Values c and ab are always negative
> - **State doesn't diffuse out of proximity of the equilibrium state $[\bar{s}\ \bar{n}_w]^T$**

Dynamics toward the equilibrium state



Non-asymptotic stability (not work well)
All real parts of λ aren't negative



Asymptotic stability (work well)
All real parts of λ are negative

$d\vec{x}/dt = A\vec{x}$
Linearly converting
$d\vec{z}/dt = A\vec{z} = \vec{\lambda}\vec{z}$
$\lambda_i = \alpha_i + j\beta_i$
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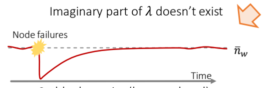
Imaginary part of λ doesn't exist



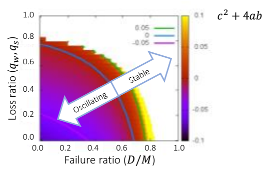
Stable dynamics (low overhead)

Imaginary part of λ exists

Oscillating dynamics (high overhead)

Stable or oscillating dynamics (1/2)

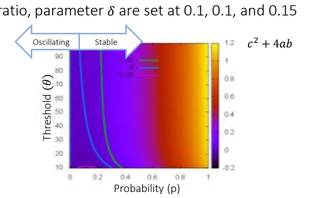
- Analyze parameter area where dynamics is stable ($c^2 + 4ab > 0$)
- Parameters p, θ, and δ are set at 0.1, 10, and 0.15



> - Figure shows that imaginary part exists ($c^2 + 4ab < 0$) in huge area
> - Dynamics of response threshold model basically oscillates

Stable or oscillating dynamics (2/2)

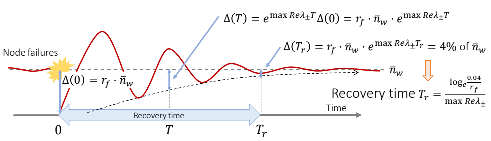
- Analyze influence of tunable parameters p and θ on value $c^2 + 4ab$
- Failure ratio, loss ratio, parameter δ are set at 0.1, 0.1, and 0.15



> - Figure shows that the dynamics is more stable as probability p is larger
> - However, existence condition $(1-D/M)(1-q_w)-\delta(1+p) > 0$ of $[\bar{s}\ \bar{n}_w]^T$ implied that robustness simultaneously deteriorated

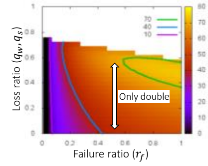
Recovery time from individual failures (1/3)

- Failure scenario
  - Failure ratio r_f represents magnitude of individual failures
  - $r_f \cdot \bar{n}_w$ workers are removed in the equilibrium state $[\bar{s}\ \bar{n}_w]^T$
- Recovery time T_r
  - Necessary time for $\Delta = \bar{n}_w - n_w$ to decrease to 4% of $\bar{n}_w$



Recovery time from individual failures (2/3)

- Analyze dependence of recovery time on loss ratio and failure ratio
- Parameters p, θ, and δ are set at 0.1, 10, and 0.15

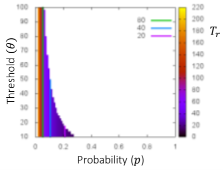


$$T_r = \frac{\log_e \frac{0.04}{r_f}}{\max Re\,\lambda_\pm}$$

> - Recovery time is within twice as far as loss ratio is higher
> - **Response threshold model is robust against information loss**

Recovery time from individual failures (3/3)

- Analyze influence of tunable parameters p and θ on recovery time
- Failure ratio, loss ratio, parameter δ are set at 0.5, 0.0, and 0.15



$$T_r = \frac{\log_e \frac{0.04}{r_f}}{\max Re\, \lambda_{\pm}}$$

> To make recovery time shorter,
> larger probability p and smaller threshold θ are better

Conclusion and future work

- Conclusion
  - Constructed the analytical model considering loss and failure
  - Investigated the influence of loss on recovery time from failure
  - Showed response threshold model is robust against information loss
- Future work
  - Suppress oscillation of the dynamics in the transient phase
  - Analyze distribution of the number of workers

Thank you!!