Master's Thesis

Title

Structural reduction method for obtaining effective attractors in noise-induced VNT control

Supervisor

Professor Masayuki Murata

Author

Toshihiko Ohba

February 10th, 2015

Department of Information Networking

Graduate School of Information Science and Technology

Osaka University

Master's Thesis

Structural reduction method for obtaining effective attractors in noise-induced VNT control

Toshihiko Ohba

Abstract

Changes in environments surrounding the Internet cause large fluctuations of traffic demands in recent years. One of approaches to achieving flexibility to accommodate changing traffic demands is to deploy a flexible infrastructure, such as WDM (Wavelength Division Multiplexing)-based networks, that can reconfigure a virtual network in a dynamical manner. Many researches have investigated methods to carry IP traffic over WDM networks. One approach to accommodating IP traffic on a WDM network is to construct a virtual network topology (VNT) and reconfigure a VNT following traffic changes. Our research group has proposed a VNT control method that is adaptive to traffic changes. The VNT control method is based on a dynamical system, called attractor selection, which models behavior where living organisms adapt to unknown changes in their surrounding environments and recover their conditions. Our VNT control method searches for a solution, i.e., a VNT that can accommodate IP traffic, using noise. The search for a solution is guided by attractors rather than made just randomly by noise. Attractors are a part of equilibrium points in the solution space. In our VNT control method, attractors correspond to VNT candidates. One of important things of our VNT control method is how to design attractors because attractors define attractive states of the VNT control. In our previous work, we prepared VNT candidates in a random manner. However, when we do not design VNT candidates properly, it takes a long time for our VNT control method to find a solution. Therefore, we propose a method to design VNT candidates in this paper. Our basic approach prepares VNT candidates whose bottleneck links (lightpaths) are different to each other. However, our exhaustive algorithm based on this approach has a problem that it takes a heavy computational time for large-scaled networks. We therefore propose a method that hierarchically contracts a network topology so that we can apply our algorithm to large-scaled networks. Evaluation results show that the VNT candidates prepared by our method can suppress maximum link utilization than the ones prepared in a random manner. As a

result, our VNT control method achieves a shorter convergence time. That is, our VNT control method becomes more adaptive to traffic changes.

Keywords

Attractor Selection

Virtual Network Topology

VNT Control

Wavelength Division Multiplexing

Wavelength-routed Network

Contents

List of Figures

List of Tables

1 Introduction

Environments surrounding the Internet have changed in recent years. Thanks to the advancement of personal Internet-enabled devices, many people have come to utilize the Internet. They benefit from various services through the Internet, new Internet services such as video on demand, cloud computing services and social network services (SNS) emerge one after another. Such changes in the Internet cause large fluctuations of traffic demands. In response to such situations, it has been required for network operators that a network has flexibility to accommodate changing traffic demands.

One of approaches to achieving the flexibility is to deploy a flexible infrastructure that can reconfigure the connectivity of network equipment and/or bandwidth allocation in a dynamical manner. Then, network operators conduct traffic engineering over the flexible infrastructure so that large fluctuations of traffic demands can be absorbed. An example of the flexible infrastructure is wavelength-routed networks based on wavelength division multiplexing (WDM) technology. Many researches have investigated methods to accommodate IP traffic over WDM networks [1–4]. IP over WDM network consists of two layers, WDM network and IP network (Fig. 1). In the WDM network, optical cross-connects (OXCs) are interconnected by optical fibers. A set of optical channels, called lightpaths, are established between IP routers via OXCs. Lighpaths and IP routers form a virtual network topology (VNT) and it accommodates IP traffic on the WDM network. IP packets as electric signals are converted into optical signals and OXCs switch optical signals in the WDM network. When fluctuations of traffic demands cause traffic congestion temporarily, control of a VNT, i.e., reconfiguration of a VNT following traffic changes, solves the traffic congestion, so that a VNT can accommodate changing traffic demands.

Many researches have devoted to develop methods to accommodate IP traffic on a VNT following traffic changes [5–14]. For example, Refs. [5–12] propose methods to configure a VNT by solving a mixed integer linear program (MILP), which aims to minimize maximum link utilization of a VNT, packet delay, or the number of resources that form a VNT such as wavelengths and router ports. A method proposed in Refs. [13] focuses on one-day traffic changes and dynamically adapts to traffic changes at a peak period. In short, these methods proposed in Refs. [5–13] use information of traffic demand matrices. However, it generally takes a long time to retrieve information of traffic demand matrices. Thus, these methods proposed in Refs. [5–13] reconfigure a

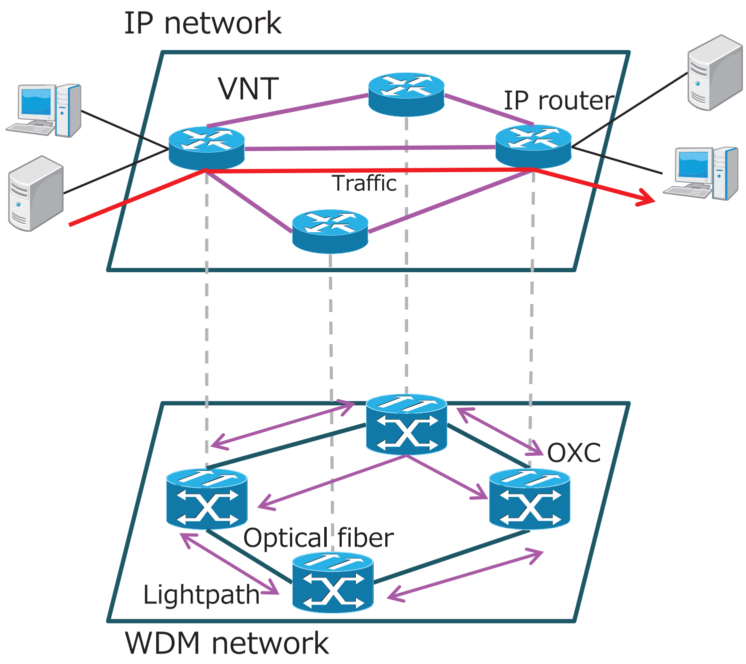

Figure 1: IP over WDM network

VNT on the basis of long-term measurement of traffic demands.

However, when traffic demands fluctuate rapidly, it is difficult for a method that designs a VNT on the basis of traffic demand matrices to reconfigure a VNT following traffic changes. For example, Refs. [15–17] point out that a flash crowd of traffic becomes more likely to occur recently. The flash crowd is the phenomenon that rapid increase of traffic to a certain web server occurs within a short time. A traffic engineering method that dose not retrieve information of traffic demand matrices is proposed in Refs. [14]. This method estimates a traffic demand matrix by measuring an amount of traffic in a short period and reconfigures a VNT. However, estimation errors in traffic demand matrices are unavoidable in general when traffic demands fluctuate largely. As a result, the method based on traffic estimation does not always reconfigure a VNT that can accommodate changing traffic demands. Therefore, it is important to achieve a method to adaptively reconfigure a VNT against traffic changes within a short time.

Our research group has proposed a VNT control method that is adaptive to traffic changes and accommodates IP traffic effectively [18, 19]. This method is based on a dynamical system, called attractor selection model, which models behavior where living organisms adapt to unknown changes in their surrounding environments and recover their conditions. In our VNT control method, attractors, which are a part of equilibrium points in the solution space, correspond to VNT candidates. The basic mechanism of VNT control consists of deterministic behavior and stochastic behavior, which are controlled by feedback that indicates conditions of an IP network. That is, our VNT control method collects not traffic demand matrices but conditions of a network, and controls a VNT based on the feedback. When traffic demands fluctuate largely, our VNT control method searches for a solution, i.e., a VNT that can accommodate traffic demands. The search for a solution is not made just randomly by the stochastic behavior but guided to attractors by the deterministic behavior. We have shown in Refs. [18, 19] that this method can reconfigure a VNT adaptively against fluctuations in network environments such as traffic changes and node failure.

In our VNT control method based on attractor selection, it is crucial to design attractors properly. This is because that attractors define attractive states of the VNT control. In Refs. [18, 19], we have prepared VNT candidates in a random manner. However, when we do not design VNT candidates properly, it takes a long time for our VNT control method to find a solution. For example, assuming that VNT candidates are tuned for only certain patterns of traffic demands, our VNT control method may not immediately reach a solution against unknown traffic changes since the search for a solution is guided by the attractors. Thus, a challenge of our VNT control method is how to design attractors under fluctuations in network environments. One extreme approach is to prepare all VNT candidates as attractors. However, the number of VNT candidates that can be kept as attractors is limited to 10～15% of the number of possible lightpaths according to the property of Hopfield Network [20]. Therefore, we newly propose a method to design VNT candidates. Our approach classifies various VNT candidates into groups on the basis of their characteristics and selects an attractor from each group. However, our exhaustive algorithm based on this approach has a problem that it takes a heavy computational time for large-scaled networks. Therefore, we also propose a method to hierarchically contract a network topology so that we can apply our algorithm to large-scaled networks. By preparing a limited number of VNT candidates that can accommodate various traffic demands, various kinds of VNTs are searched by attractor selection. As a result, our VNT control method can find a solution within a short period. That is, our VNT control method becomes more adaptive to traffic changes.

The rest of this paper is organized as follows. In Section 2, we explain our VNT control method based on attractor selection. We then propose a method to design VNT candidates and evaluate our algorithm in Section 3. We also propose a method to hierarchically design VNT candidates for large-scaled networks in Section 4. In Section 5, we evaluate our method that hierarchically designs VNT candidates and our VNT control method using VNT candidates obtained by our method as attractors. We conclude this paper in Section 6.

2 VNT Control Based on Attractor Selection

In this section, we explain the VNT control method based on attractor selection proposed in [18, 19]. In this paper, traffic flows between IP routers on a shortest path of a VNT. We simply refer to link utilization on a VNT as link utilization.

2.1 Overview of VNT Control Based on Attractor Selection

A dynamic system driven by attractor selection model adapts to unknown changes in its surrounding environments [21]. In attractor selection model, attractors are a part of equilibrium points in the solution space where the system conditions are preferable. A basic mechanism of attractor selection model consists of deterministic behavior and stochastic behavior. The behavior of the dynamic system driven by attractor selection is described as follows:

$$\frac{d\mathbf{x}}{dt} = \alpha \cdot f(\mathbf{x}) + \eta. \tag{1}$$

A state of the system is represented as $\mathbf{x} = (x_1, ..., x_i, ..., x_n)$ (n is the number of state variables). $f(\mathbf{x})$ represents the deterministic behavior and η represents the stochastic behavior. The behavior is controlled by activity α, which is simple feedback of the system conditions. When the current system conditions are suitable for the environment and α takes a large value, the deterministic behavior drives the system to the attractor. When the current system conditions are poor, i.e., when α takes a small value, the stochastic behavior is dominant in controlling the system. While the stochastic behavior dominates over the deterministic behavior, the state of the system fluctuates randomly due to noise η and the system searches for a solution where the system conditions are preferable. In this way, attractor selection adapts to environmental changes using properly both the deterministic behavior and the stochastic behavior based on the activity.

Our VNT control method considers the state of the system $\mathbf{x}$ as a state of all possible lightpaths that form a VNT and uses the conditions of the IP network as the activity. Our VNT control method then configures a VNT so that the comfort of the IP network gets improved when the conditions of the IP network become uncomfortable due to fluctuations of traffic demands.

2.2 Dynamics of VNT Control

Our VNT control method decides whether or not to set up a lightpath l_i based on a state variable $x_i(\in \mathbf{X})$. Dynamics of the state variable x_i is defined as

$$\frac{dx_i}{dt} = \alpha \cdot \left(\varsigma\left(\sum_j W_{ij} x_j \right) - x_i \right) + \eta. \tag{2}$$

Activity α indicates the conditions of the IP network. The term $\varsigma(\Sigma_j W_{ij} x_j) - x_i$ represents the deterministic behavior. $\varsigma(z) = \tanh(\frac{\mu}{2} z)$ is a sigmoidal regulation function. The first term is calculated using a regulatory matrix W_{ij}. The second term η represents the stochastic behavior and is white Gaussian noise. After x_i is updated on the basis of Eq. (2), we decide whether or not to set up the lightpath l_i. Specifically, we set a threshold to zero and if x_i is equal to or more than the threshold, we set up the lightpath l_i and otherwise tear down the lightpath l_i.

2.2.1 Activity

Our VNT control method uses maximum link utilization on the IP network as a performance metrics. Although it is necessary to collect load information on all links (lightpaths) in the IP network, this can be retrieved in much shorter time than traffic demand matrices used by existing VNT control methods. We convert the maximum link utilization on the IP network, u_{max}, into the activity α using the following expression Eq. (3). The activity is in a range [0, γ]. The constant number θ is a threshold for the VNT control. When the maximum link utilization is more than the threshold θ, the activity rapidly approaches to zero and our VNT control method searches for a solution so that the comfort of the IP network gets improved. δ is also a constant number, which determines an inclination of the function.

$$\alpha = \frac{\gamma}{1 + \exp(\delta \cdot (u_{max} - \theta))} \tag{3}$$

2.2.2 Regulatory Matrix

We set the regulatory matrix so that it has a set of VNT candidates as attractors. That is, we set the regulatory matrix $\mathbf{W}$ so that $d\mathbf{x}/dt$ in Eq. (1) is equal to zero when a VNT reconfigured by our VNT control method $\mathbf{x} = (x_1, ..., x_i, ..., x_n)$ is one of attractors. In order to store attractors in the regulatory matrix, we use a method to decide the regulatory matrix using the pseudo inverse matrix, which is shown in Refs. [22]. Specifically, assuming that we set m VNT candidates as attractors and one of the candidates is represented as $\mathbf{x}^{(k)} = (x_1^{(k)}, ..., x_i^{(k)}, ..., x_n^{(k)})(1 \leq k \leq m)$, the regulatory matrix that has m attractors is calculated as

$$\mathbf{W} = \mathbf{X}^{+}\mathbf{X}, \tag{4}$$

where $\mathbf{X}$ is a matrix that has $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, ..., \mathbf{x}^{(m)}$ in each row and $\mathbf{X}^{+}$ is the pseudo inverse matrix of $\mathbf{X}$.

3 Design Method of Attractors

3.1 Design Problem of Attractors

We suppose that we design attractors, i.e., VNT candidates, for a network with n nodes. Although the size of the solution space is 2^{n^2}, the number of VNT candidates that can be kept as attractors is limited to $10 \sim 15\%$ of the number of possible lightpaths n^2 [20]. Moreover, VNT candidates should have diversity so that they can adapt to various fluctuations of traffic demands. Therefore, a problem to properly design VNT candidates as attractors comes down to the problem to select $0.1n^2$ VNT candidates with diversity from the solution space.

For the problem, we focus on characteristics of VNT candidates. Since a VNT configured by our VNT control method based on attractor selection finally converges on one of attractors, any of the attractors should accommodate the current traffic demand. In other words, each VNT candidate should accommodate different traffic demand to each other. That is, it is important that each attractor has different characteristics to each other, which produces diversity of VNT candidates. Therefore, we take an approach to classifying VNT candidates into groups on the basis of their characteristics and selecting an attractor from each VNT candidates group. By preparing a limited number of VNT candidates with diverse characteristics, various kinds of VNTs are searched by attractor selection. As a result, our VNT control method finds a solution within a short time. That is, our VNT control method becomes more adaptive to traffic changes.

3.2 Exhaustive Algorithm to Design Attractors with Different Characteristics

We develop an algorithm that selects attractors for our VNT control method. The goal of our algorithm is to select $0.1n^2$ attractors with diversity from 2^{n^2} solution space. An outline of our algorithm is as follows.

(1) Enumerate isomorphic VNT candidates of VNT g.

(2) Classify the enumerated VNT candidates on the basis of their characteristics.

(3) Select an attractor from each group of the VNT candidates.

In our algorithm, a VNT g is given in advance. We use a heuristic method to configure the VNT g with a traffic demand matrix T. Note that although we use the traffic demand matrix T to design VNT candidates, we do not use T for our VNT control method. We explain the detail of the algorithm below.

3.2.1 Enumeration of VNT candidates

We enumerate isomorphic VNTs of g. The isomorphic VNTs are generated by exchanging all the nodes of the VNT g. Fig. 2 illustrates examples of isomorphic VNTs. In Fig. 2, the VNT g_1 consists of five nodes $N0, N1, ..., N4$ and the VNT g_2 and g_3 is one of isomorphic VNTs of g_1. The isomorphic VNT g_2 is generated by shifting $N0$ of the VNT g_1 to $N1$, $N1$ to $N2$, $N2$ to $N3$, $N3$ to $N4$, $N4$ to $N0$. The isomorphic VNT g_3 is generated by shifting $N0$ of the VNT g_1 to $N4$, $N4$ to $N3$, $N3$ to $N2$, $N2$ to $N1$, $N1$ to $N0$. However, VNT candidates that do not meet restrictions on resources in a physical network, such as the number of router ports each node has, are excluded. Thus, the number of the enumerated VNT candidates is $n!$ at most.

In Fig. 2, we assume that a VNT g_1 is configured by a heuristic method with a traffic demand matrix T_1 and a traffic load on the red colored link between the nodes $N3$ and $N4$ is highest. Since the VNT g_1 is configured by a heuristic method with the traffic demand matrix T_1, the VNT g_1 can accommodate T_1. Let us assume that a traffic demand matrix T_2 is generated by exchanging all the elements of T_1 . Since we exchange the nodes of the VNT g_1 in order to generate the isomorphic

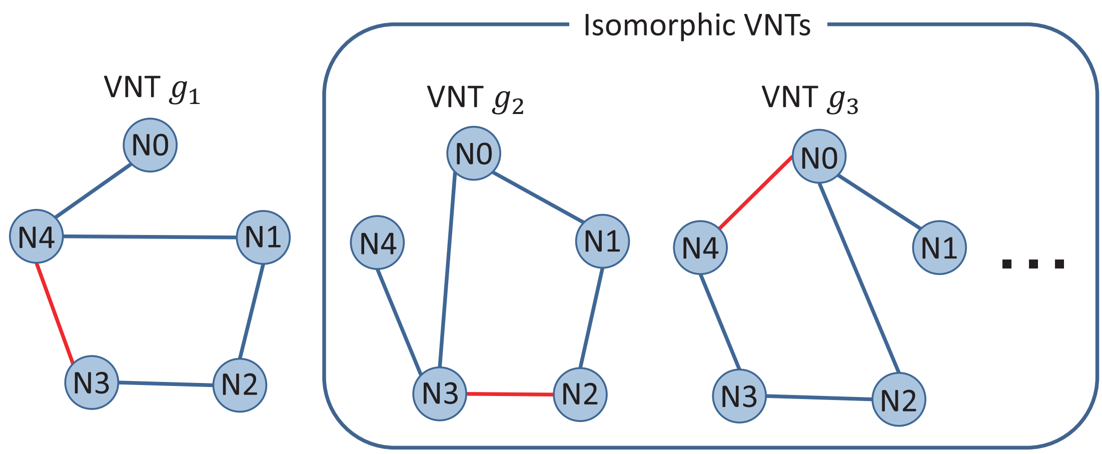

Figure 2: Examples of isomorphic VNTs

VNT g_2, a traffic load on the link between the nodes $N2$ and $N3$ is highest and the VNT g_2 can accommodate T_2. That is, it is expected that any of the isomorphic VNTs can accommodate changing traffic demands, unless each value of a traffic demand matrix is too large. Hereafter, we denote G as the set that includes the VNT g and the enumerated VNT candidates.

3.2.2 Classification of the VNT candidates

We classify the VNT candidates that belong to G into groups on the basis of their characteristics. We use Edge Betweenness Centrality, which is the number of shortest paths that go through the link, as characteristics of VNT candidates. Then, we classify the VNT candidates that have different bottleneck links each other into different groups, as shown in Fig. 3. A bottleneck link has the largest value of Edge Betweenness Centrality among links that form a VNT candidate. When each VNT candidate selected as an attractor has a different bottleneck link, it is expected that any of VNT candidates selected as attractors accommodates various traffic demands. Note that in our VNT control method based on attractor selection, the maximum link utilization indicates the comfort of the IP network. It is likely that a link whose link utilization is high has a high value of Edge Betweenness Centrality. Therefore, we classify the VNT candidates that have the same bottleneck links into the same group. A formal definition of the VNT candidates group is below.

- $p = (s, d)$: an identifier of a node pair, a source node s and a destination node d

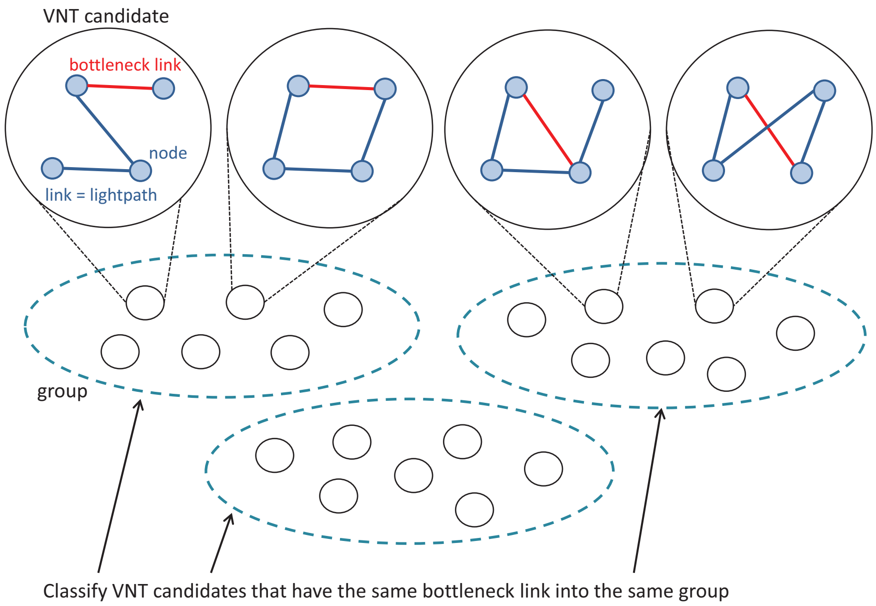

Figure 3: Classification of VNT candidates

- l_p : a link (lightpath) established between the node pair p
- $C(g_i, l_p)$: a value of Edge Betweenness Centrality of the link l_p in a VNT candidate g_i

The VNT candidates group G_p that is expected to have the bottleneck link l_p is described as

$$G_p = \{g_i | g_i \in G, C(g_i, l_p) = \max_q C(g_i, l_q)\}, \tag{5}$$

using the above notations.

In this way, we divide the set of the VNT candidates enumerated in Section 3.2.1. The number of groups is at most n^2 since the number of possible lightpaths is n^2. However, because the number of VNT candidates that can be kept as attractors is $0.1n^2$, we further merge the VNT candidates groups.

We merge the VNT candidates groups with a condition when traffic loads of their bottleneck links are highly correlated. The condition is satisfied when a correlation of traffic loads is high between two links connected via a node whose degree is low. Fig. 4 illustrates the condition with which we merge the VNT candidates groups. When traffic flows from a source node s to a destination node d via a node a whose degree is low, a part of traffic that flows on a link $l_{(s,a)}$

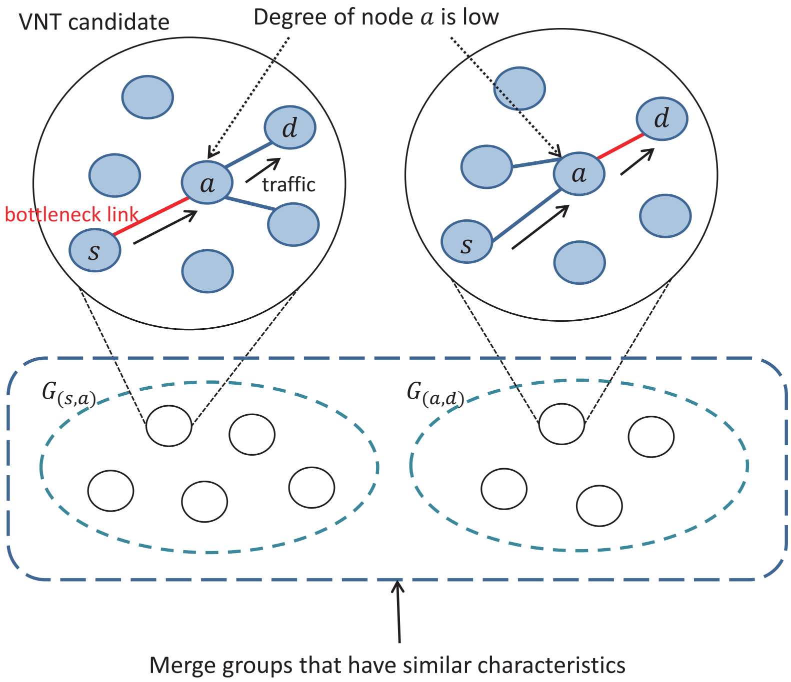

Figure 4: Merger of VNT candidates groups

also flows on a link $l_{(a,d)}$. That is, if the link $l_{(s,a)}$ is a bottleneck link, it is likely that a traffic load on the link $l_{(a,d)}$ is also high. Therefore, we consider that VNT candidates that belong to a group $G_{(s,a)}$ and $G_{(a,d)}$ have similar characteristics. Based on this heuristic, we merge the VNT candidates groups as

$$G_{(s,d)} \leftarrow G_{(s,a)} \cup G_{(a,d)} \cup G_{(s,d)}, \tag{6}$$

where the degree of the node a is low. In Eq. (6), we also regard VNT candidates included in the group $G_{(s,d)}$, which have a bottleneck link $l_{(s,d)}$, have similar characteristics to the group $G_{(s,a)}$ and $G_{(a,d)}$. The reason is that it is likely that a traffic load on the link $l_{(s,d)}$ is high when the link $l_{(s,a)}$ and $l_{(a,d)}$ are bottleneck links. We select the node a, s and d in ascending order of the degree, since a correlation of traffic loads on the link $l_{(s,a)}$ and $l_{(a,d)}$ is high when the degree of the node a is low. However, since each group has different VNT candidates, we select the node a, s and d on the basis of the average value of the degree among all the VNT candidates in the group. We repeatedly merge the VNT candidates groups until the number of VNT candidates groups becomes about $0.1n^2$.

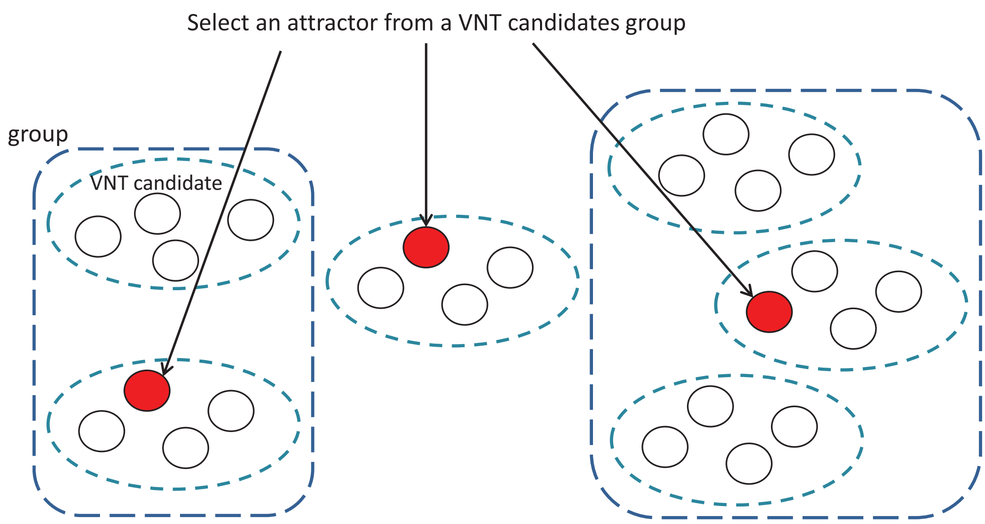

Figure 5: Selection of attractors from VNT candidates groups

3.2.3 Selection of attractors from VNT candidates groups

We finally select an attractor from each of the VNT candidates group, as shown in Fig. 5. We select a VNT candidate as an attractor whose maximum value of Edge Betweenness Centrality is lowest among the VNT candidates group, since the smaller the value of Edge Betweenness Centrality is, the more likely it is that the maximum link utilization is suppressed.

3.3 Evaluation of Effect of Design Approach: Engineered or Random

In this section, we evaluate performance of VNT candidates obtained by the algorithm described in Section 3.2. That is , we evaluate effect of our approach to designing attractors. We use a 10-nodes network that has a ring topology as a physical network topology. Each node has five router ports, i.e., five transmitters and five receivers. We configure a VNT candidate using I-MLTDA [23] as a heuristic method with a traffic demand matrix whose elements follow a log-normal distribution. We obtain 10 VNT candidates by following the algorithm in Section 3.2. For the evaluation, we use 1,000 patterns of traffic demands between each node pair according with a log-normal distribution. We compare a method that constructs a VNT candidate by establishing lightpaths between randomly chosen node pairs. This is because that we design VNT candidates in a random manner in Refs. [18, 19]. The number of VNT candidates is the same for all methods.

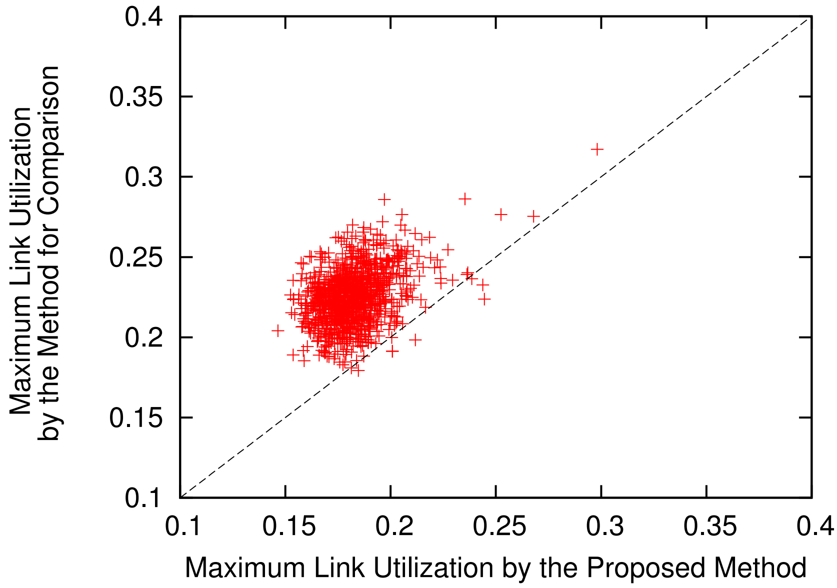

Figure 6: Distribution of maximum link utilization: 10-nodes, 5-ports

Fig. 6 shows distribution of maximum link utilization for each traffic pattern. This figure shows the lowest value of the maximum link utilization among the VNT candidates for each traffic pattern. The horizontal axis shows the maximum link utilization of the VNT candidates by our method and the vertical axis shows that of the VNT candidats by the method for comparison. In Fig. 6, we can see that the maximum link utilization of the VNT candidates by our method is low against more traffic patterns than the other method. Specifically, the VNT candidates by our method make the maximum link utilization lower than the ones by the other method against 991 traffic patterns. That is, our algorithm can design VNT candidates that suppress traffic loads against various traffic demands, compared with the other method. Thus, we can design better attractors based on our approach, which classifies VNT candidates into groups on the basis of their characteristics and selects an attractor from each group.

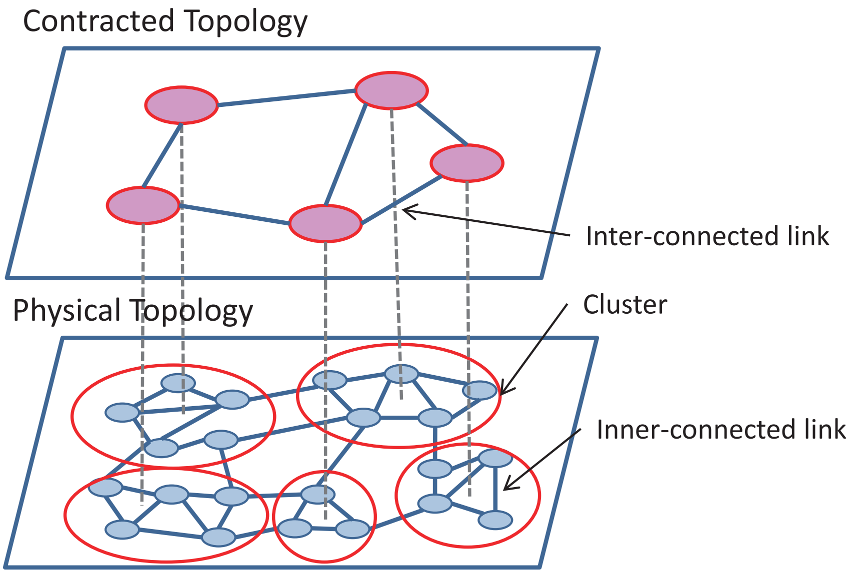

Figure 7: Contraction of physical network topology

4 Structural Reduction Method for Designing Attractors

4.1 Scalability Problem of Exhaustive Algorithm

Although we can design better attractors based on our approach, as shown in Section 3.3, the algorithm described in Section 3.2 has a problem that it takes a heavy computational time for large-scaled networks. This is because that the number of enumerated VNT candidates increases explosively as the number of nodes n increases. With our ordinal PC, we can decide VNT candidates for up to 10-nodes networks with a realistic calculation time. Therefore, we take an approach to contracting a physical network topology and applying the algorithm in Section 3.2 to the contracted network topology. Specifically, we divide a physical network topology into clusters so that each cluster has several nodes, as shown in Fig. 7. We reduce the number of nodes in a network topology by considering the clusters as nodes, and apply the algorithm in Section 3.2 to the contracted network topology.

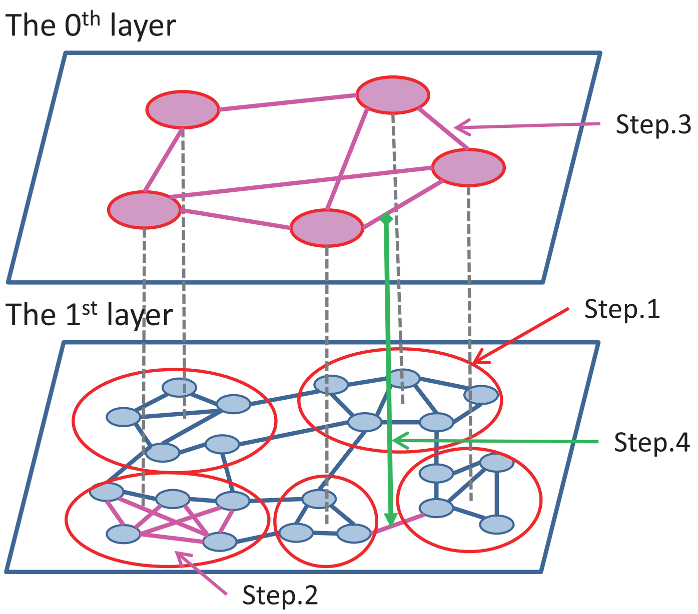

Figure 8: Outline of the method to design attractors in a hierarchical manner

4.2 Algorithm to Design Attractors in a Hierarchical Manner

An outline of our method to design VNT candidates in a hierarchical manner is as follows. The outline is also illustrated in Fig. 8.

Step.1 Divide a physical network topology into clusters and decide clusters at multiple layers.

Step.2 Construct VNT candidates in clusters at the bottom layer

Step.3 Construct VNT candidates at upper layers by following the algorithm in Section 3.2.

Step.4 Connect lightpaths between clusters to nodes in the clusters.

We explain the detail of the algorithm below.

Step.1 Cluster division of a physical network topology

We divide a physical network topology into c clusters. When the number of vertexes in a cluster is more than c, we divide the cluster into clusters recursively until the number of vertexes in a cluster is equal to or less than c, i.e., we decide clusters at multiple layers. An upper layer consists of clusters that have nodes at a lower layer. For example, in a three-layer network, the top layer consists of clusters that have nodes at the middle layer. Then, nodes at the middle layer are clusters that have nodes at the bottom layer (Fig. 11).

When the number of nodes in a physical network topology is n and the average of the number of vertexes in a cluster at each layer is c, we can consider that the physical network topology has $\lceil \log n / \log c \rceil$ layers.

Step.2 Construction of VNT candidates inside clusters at the bottom layer

We construct VNT candidates inside clusters at the bottom layer. We construct a VNT candidate that has a full-mesh topology or a star topology with several hub nodes in a cluster at the bottom layer. This is because a cluster can adapt to traffic changes in a cluster and can keep connectivity when network failure occurs.

Step.3 Construction of VNT candidates at upper layers

We design VNT candidates with diversity at upper layers by following the algorithm in Section 3.2. However, we do not need to merge VNT candidates groups. We prepare $c(c-1)/2$ VNT candidates groups at most, since we set up lightpaths bidirectionally. This is because we do not need to set up single directional lightpaths in order to decide a limited number of VNT candidates with diverse characteristics. We construct VNT candidates at upper layers as follows.

Step.3-1 Calculate a VNT candidates using a heuristic method and enumerate isomorphic VNT candidates of it.

Step.3-2 Classify the enumerated VNT candidates into $c(c-1)/2$ groups at most on the basis of Edge Betweenness Centrality.

Step.3-3 Select an attractor from each group of the VNT candidates.

Step.4 Connection between clusters

We connect lightpaths between clusters to nodes in the clusters. That is, we map lightpaths between clusters at upper layers to lightpaths between nodes inside the clusters at lower layers. We establish lightpaths between clusters from the kth layer to the $k+1$ layer, i.e., from an upper layer to a lower layer. We establish lightpaths between nodes in clusters as follows.

- C_x^k : the xth cluster at the kth layer
- V_x^k : nodes that belong to C_x^k
- $l_{i,j}^k$: a lightpath bidirectionally established between C_i^k and C_j^k
- k_u : the number of lightpaths connected to a node u (the degree of the node u)

The probability we establish the lightpath $l_{i,j}^k$ between $u(\in V_i^k)$ and $v(\in V_j^k)$ is described as

$$P_{u,v} = (k_u k_v)^{-1}. \tag{7}$$

Eq. (7) intends to balance traffic loads. Since it is likely that a larger amount of traffic flows via a node as the degree of the node increases, we connect nodes whose degree is low.

5 Evaluation of Structural Reduction Method

5.1 Performance of VNT Candidates Obtained by Structural Reduction Method

In this section, we evaluate performance of VNT candidates by the structural reduction method in Section 4. Here, we use a 25-nodes network and each node has 10 router ports. We consider the two-layer network so that every cluster has five nodes, as shown in Fig. 9. A topology at the top layer is regarded to be composed of five nodes with three router ports and we obtain seven VNT candidates by following Step.3 of our method. The reason why the number of VNT candidates is seven is that the enumerated VNT candidates are classified into seven groups, i.e., only seven lightpaths become bottleneck links among the enumerated VNT candidates. Thus, we do not merge the VNT candidates groups. A VNT candidate in each cluster at the bottom layer has a full-mesh topology. When a lightpath is established between two nodes at the top layer, five bidirectional lightpaths are established between the corresponding clusters at the bottom layer. In this way, we connect seven VNT candidates at the top layer and one VNT candidate at the bottom layer. Finally, we obtain seven VNT candidates.

Table 1 shows the number of ports used at each layer in the VNT candidates. At the top layer, ports are used to establish lightpaths between nodes that belong to different clusters to each other.

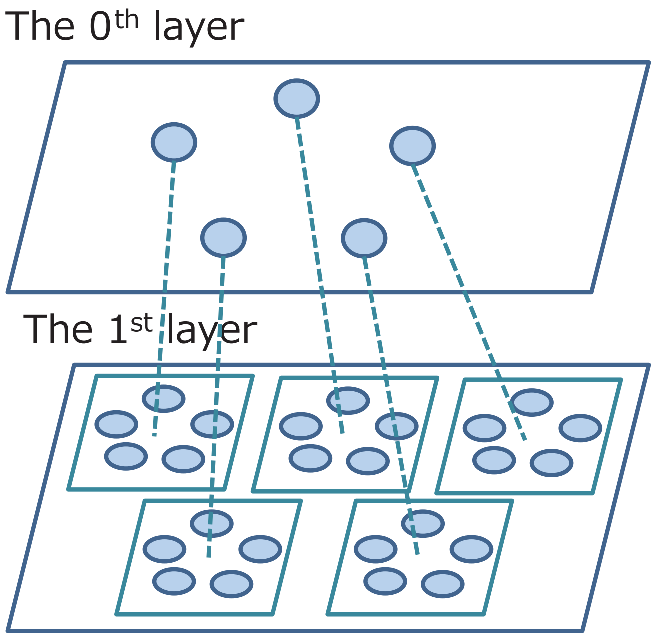

Figure 9: Clusters in a 25-nodes network

Table 1: Number of ports used at each layer: 25-nodes network

Layer	Number of ports
0th layer	140
1st layer	100

At the bottom layer, ports are used to establish lightpaths between nodes inside the same clusters. We can see that the number of ports at the top layer is 1.4 times as that of the bottom layer.

For the evaluation, we use 1,000 patterns of traffic demands between each node pair according with a log-normal distribution. We compare a method that constructs a VNT candidate by establishing lightpaths between randomly chosen node pairs. This is because that we design VNT candidates in a random manner in Refs. [18, 19]. The number of VNT candidates is the same for all methods.

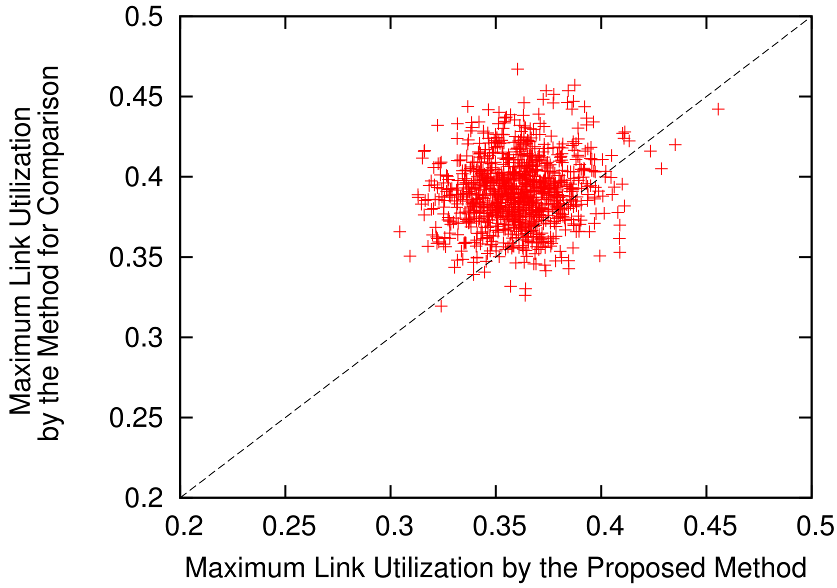

Figure 10: Distribution of maximum link utilization: 25-nodes, 10-ports

Fig. 10 shows distribution of maximum link utilization for each traffic pattern. This figure shows the lowest value of the maximum link utilization among the VNT candidates for each traffic pattern. In the figure, we can see that our method can design VNT candidates that suppress the maximum link utilization against more traffic patterns than the other method. Specifically, the VNT candidates by our method can make the maximum link utilization lower than the ones by the other method against 877 traffic patterns. In other words, the method in Section 4 can design VNT candidates that suppress traffic loads against various traffic demands, compared with the other method. That is, the structural reduction method in Section 4 can design better VNT candidates than the other method for larger networks where we cannot apply the algorithm in Section 3.2.

Note that establishing more lightpaths between clusters than inside clusters, i.e., using more ports at the top layer than the bottom layer, leads to suppression of the maximum link utilization. Assuming that the number of lightpaths between clusters is small, traffic loads on the lightpaths are high due to traffic aggregation. When the number of lightpaths between clusters is enough, traffic demands transfered between the clusters are distributed and a traffic load on each lightpath between the clusters is reduced. That is, by establishing more lightpaths between clusters, VNT candidates obtained by our method can accommodate traffic effectively.

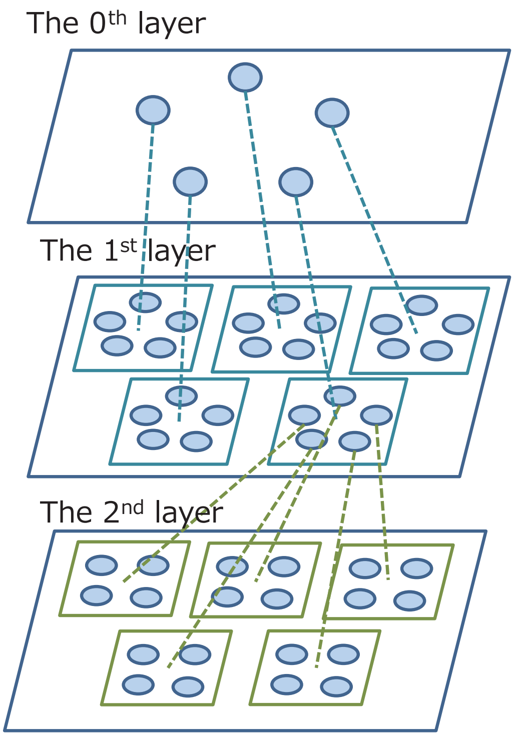

Figure 11: Clusters in a 100-nodes network

In addition, we evaluate performance of VNT candidates by our method in larger-scaled networks. Here, we use a 100-nodes network and each node has 32 router ports. We consider the three-layer network as shown in Fig. 11. A topology at the top layer and a topology in each cluster at the middle layer are considered to be composed of five nodes with three router ports, and we obtain seven VNT candidates for each layer. At the middle layer, since there are seven VNT candidates in each cluster and the number of clusters is five, the number of VNT candidates at the middle layer is 7^5, counting all combinations. Therefore, we use the same VNT candidate in all the clusters at the middle layer. That is, we decide seven VNT candidates at the middle layer. A VNT candidate in each cluster at the bottom layer has a full-mesh topology. When a lightpath is established between two nodes at an upper layer, five bidirectional lightpaths are established between the corresponding clusters at a lower layer. In this way, we connect seven VNT candidates at the top layer, seven VNT candidates at the middle layer and one VNT candidate at the bottom layer. Finally, we obtain 49 VNT candidates.

Table 2: Number of ports used at each layer: 100-nodes network

Layer	Number of ports
0th layer	1400
1st layer	700
2nd layer	300

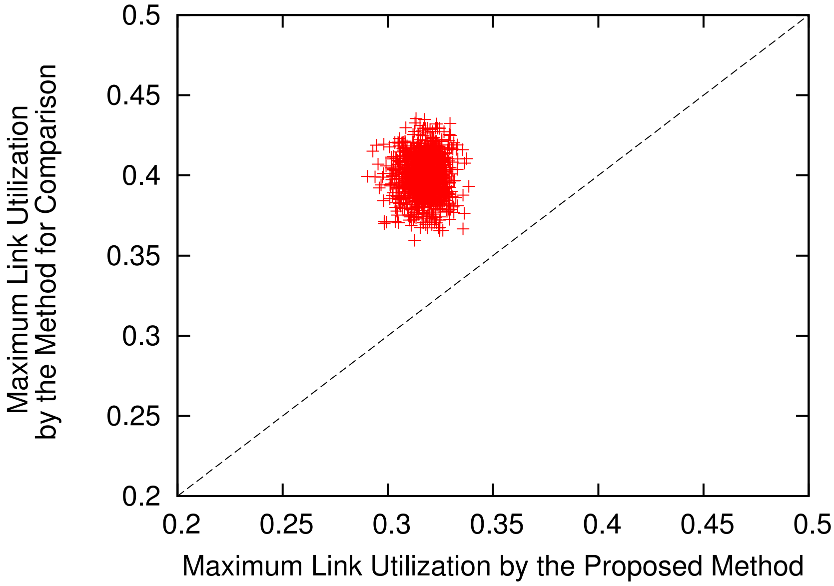

Figure 12: Distribution of maximum link utilization: 100-nodes, 32-ports

Table 2 shows the number of ports used at each layer in the VNT candidates. At the top layer, ports are used to establish lightpaths between nodes that belong to different clusters at the middle layer to each other. At the middle layer, ports are used to establish lightpaths between nodes that belong to different clusters at the bottom layer to each other. At the bottom layer, ports are used to establish lightpaths between nodes inside the same clusters. We can see that the number of ports at the middle layer is about twice of that of the bottom layer, and the number of ports at the top layer is twice of that of the middle layer.

Fig. 12 shows distribution of maximum link utilization for each traffic pattern. This figure shows the lowest value of the maximum link utilization among the VNT candidates for each traffic pattern. Traffic demands and a method for comparison are similar to those used in the evaluation

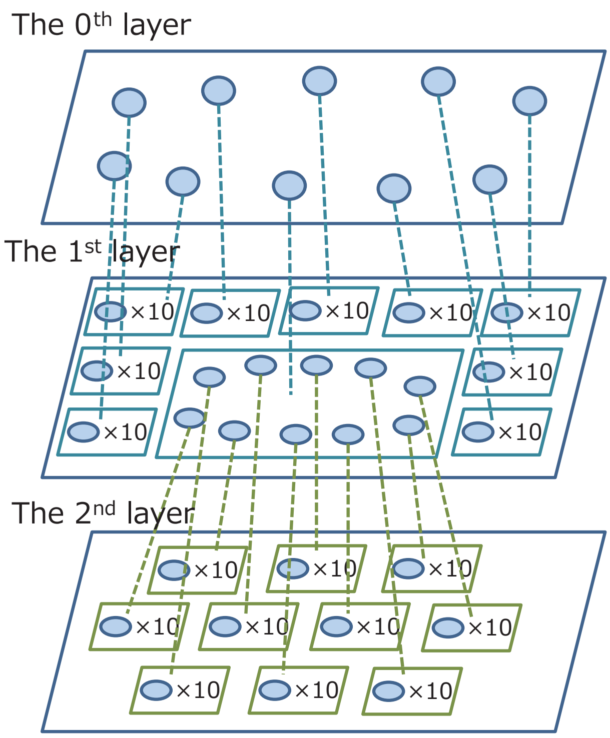

Figure 13: Clusters in a 1000-nodes network

for the 25-nodes network. However, since the total traffic demand is excessive in the 100-nodes network, we use one third of traffic demands used in the 25-nodes network. In Fig. 12, we can see that the maximum link utilization of the VNT candidates by our method is less than that of the VNT candidates by the other method against all the traffic patterns. We find that our method can design better VNT candidates for three-layer networks.

Moreover, we use a 1000-nodes network and each node has 64 router ports. We consider the three-layer network as shown in Fig. 13. A topology at the top layer and a topology in each cluster at the middle layer are considered to be composed of 10 nodes with five router ports, and we obtain 26 VNT candidates for each layer. At the middle layer, since there are 26 VNT candidates in each cluster and the number of clusters is 10, the number of VNT candidates at the middle layer is 26^{10}, counting all combinations. Therefore, we use the same VNT candidate in all the clusters at the middle layer. That is, we decide 26 VNT candidates at the middle layer. A VNT candidate in each cluster at the bottom layer has a star topology with four hub nodes. When a lightpath

Table 3: Number of ports used at each layer: 1000-nodes network

Layer	Number of ports
0th layer	44000
1st layer	14000
2nd layer	6000

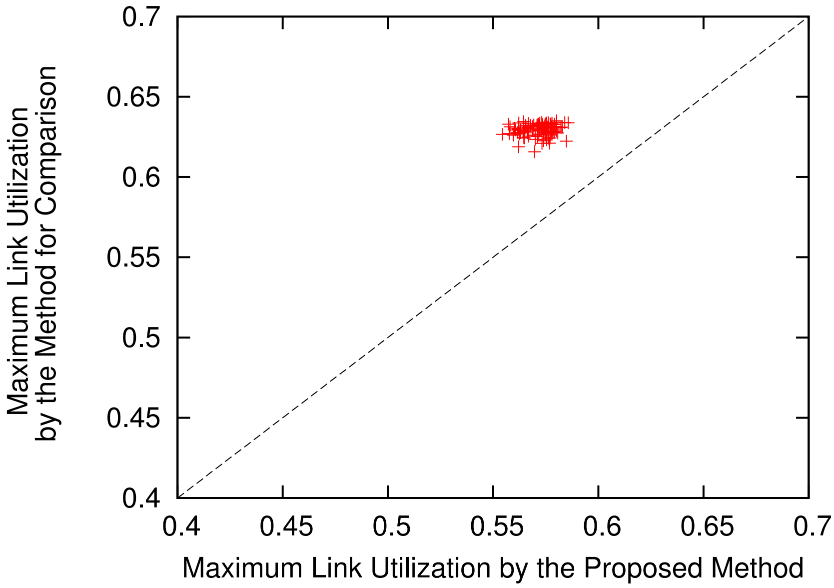

Figure 14: Distribution of maximum link utilization: 1000-nodes, 64-ports

is established between two nodes at an upper layer, 14 bidirectional lightpaths are established between the corresponding clusters at a lower layer. In this way, we connect 26 VNT candidates at the top layer, 26 VNT candidates at the middle layer and one VNT candidate at the bottom layer. Finally, we obtain 676 VNT candidates.

Table 3 shows the number of ports used at each layer in the VNT candidates. We can see that the number of ports at the middle layer is about twice of that of the bottom layer, and the number of ports at the top layer is about three times as that of the middle layer .

For the evaluation, we use 100 patterns of traffic demands between each node pair according with a log-normal distribution. However, since the total traffic demand is excessive in the 1000-nodes network, we use one sixth of traffic demands used in the 25-nodes network, i.e., a half of traffic demands used in the 100-nodes network. A method for comparison are similar to those used in the evaluation for the 25-nodes network and the 100-nodes network.

Fig. 14 shows distribution of maximum link utilization for each traffic pattern. In Fig. 14, we can see that the VNT candidates by our method make the maximum link utilization lower than the ones by the other method against all the traffic patterns. We find that our method can design better VNT candidates for 1000-nodes networks, and we believe that our method can design better VNT candidates for larger-scaled networks.

Here, we focus on the number of ports used at each layer, i.e., the number of lightpaths established at each layer, in the networks used for the evaluation. Comparing the 25-nodes network and the 100-nodes network, although the number of nodes inside clusters, i.e., cluster size, at the bottom layer is the same, the cluster size at the top layer in the latter network is larger since the latter has three layers. Then, comparing the 100-nodes network and the 1000-nodes network, although the number of layers is the same, the cluster size and the number of clusters at each layer in the latter network is larger. That is, as the number of nodes in a network increases, the cluster size and/or the number of clusters increases. The larger the cluster size is, the larger amount of traffic demands are transfered between clusters. As a result, traffic loads on lightpaths between clusters becomes higher. However, by using more ports to establish lightpaths between clusters, traffic demands transfered between clusters are distributed and a traffic load on each lightpath between clusters is reduced. Therefore, as the number of nodes in a network increases, we should use more ports, i.e., establish more lightpaths, at an upper layer. Actually, we use more ports at an upper layer than a lower layer as the number of nodes in a network increases: in the 25-nodes network, the number of ports used at the top layer is 1.4 times as that of the bottom layer, as shown in Table 1. In the 100-nodes network, the number of ports used at an upper layer is twice of that of a lower layer, as shown in Table 2. In the 1000-nodes network, the number of ports used at an upper layer is twice or three times as that of a lower layer, as shown in Table 3.

5.2 Adaptability of VNT Control Based on Attractor Selection

In this section, we evaluate adaptability of our VNT control method based on attractor selection described in Section 2, using VNT candidates obtained by our method as attractors. We set the target maximum link utilization θ in Eq. (3) to 0.5, and our VNT control succeeds when the maximum link utilization is suppressed less than 0.5 during 10 successive steps of the VNT control. We

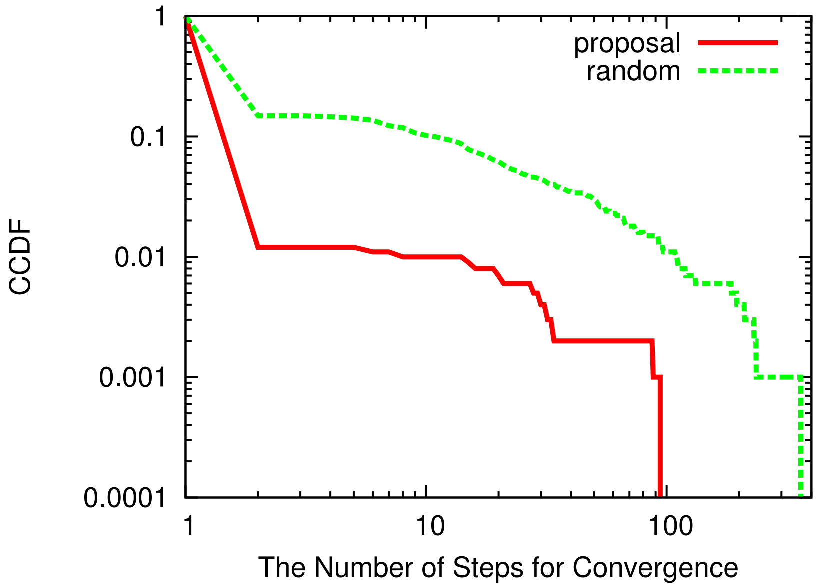

Figure 15: Distribution of the number of steps for convergence: 25-nodes, 10-ports

evaluate the number of steps of our VNT control required for success of the VNT control, i.e., the number of steps for convergence. At each step, our VNT control method collects load information on all lightpaths, calculates the activity α, and reconfigures a VNT. We set γ to 1 and δ to 50 in Eq. (3).

Fig. 15 shows distribution of the number of steps for convergence in the 25-nodes network. Traffic demands and VNT candidates used as attractors are similar to those described in Section 5.1. The horizontal axis shows the number of steps for convergence and the vertical axis shows CCDF of the number of steps. We can find that our VNT control that uses the VNT candidates by our method as attractors needs less steps for convergence. Since the VNT candidates by our method can suppress the maximum link utilization, as shown in Section 5.1, our VNT control method is guided by the better attractors and finds a solution, i.e., a VNT that can accommodate traffic, within a shorter time.

Fig. 16 shows distribution of the number of steps for convergence in the 100-nodes network. Traffic demands and VNT candidates used as attractors are similar to those described in Section 5.1. The horizontal axis shows the number of steps for convergence and the vertical axis shows CCDF of the number of steps. We can find that the VNT control that uses the VNT candidates by

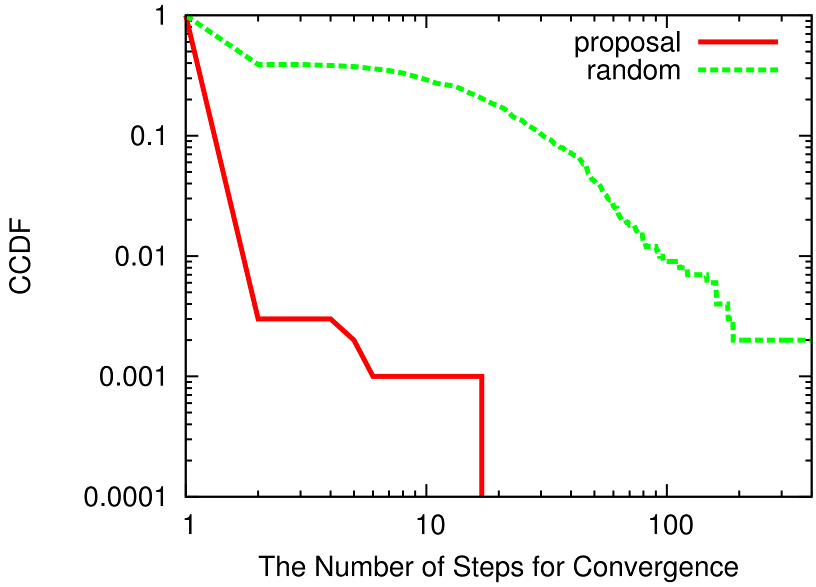

Figure 16: Distribution of the number of steps for convergence: 100-nodes, 32-ports

our method as attractors takes less steps for convergence.

The VNT control that uses the VNT candidates by our method needs about 100 steps at most in the 25-nodes network. On the other hand, it takes only about 20 steps at most in the 100-nodes network. This is because that the VNT control method can find a solution within a much shorter time in the 100-nodes network since the VNT candidates by our method can make the maximum link utilization much lower than the ones by the other method, as shown in Fig. 10 and 12.

When we evaluate the VNT control method based on attractor selection in the 1000-nodes network using the VNT candidates by our method as the attractors, it is expected that we obtain similar results to the above. This is because that the VNT candidates by our method in the 1000-nodes network can suppress the maximum link utilization than the ones by the other method, as shown in Fig. 14.

5.3 Impact of Physical Network Topology on Adaptability of VNT Control

In this section, we design VNT candidates by dividing a large-scaled network into clusters on the basis of the physical network topology. And then, we evaluate adaptability of our VNT control method using the VNT candidates as attractors. Here, we use JPN25 model and USNET as

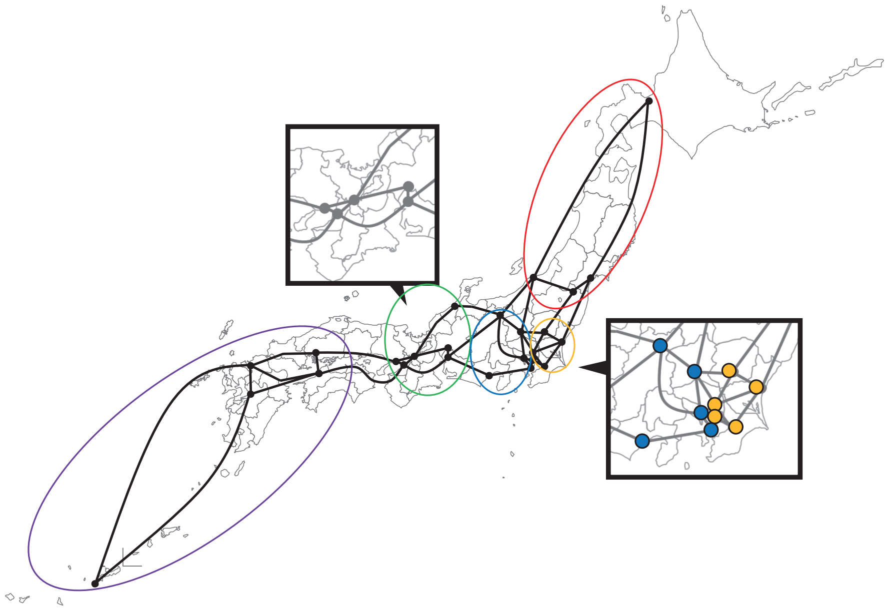

Figure 17: Clusters in JPN25 model

physical network topologies. When we design VNT candidates for the two networks, we use the structural reduction method in Section 4. We use Louvain method [24] to divide the physical networks into clusters, so that nodes inside clusters are densely connected by optical fibers and nodes that belong to different clusters are sparsely connected.

5.3.1 Evaluation for JPN25 model

JPN25 model has 25 nodes and each node has 10 ports. We divide the physical network into five clusters, as shown in Fig. 17. The nodes surrounded by a circle belong to the same cluster: one cluster has six nodes, three clusters has five nodes and one cluster has four nodes. We consider JPN model as the two-layer network, which consists of the top layer whose nodes are the clusters and the bottom layer that is equal to the original network. A topology at the top layer is regarded to be composed of five nodes with three router ports and we obtain seven VNT candidates by following Step.3 of our structural reduction method. A VNT candidate in each cluster at the bottom layer has a star topology with two hub nodes. When a lightpath is established between two nodes at the top layer, five bidirectional lightpaths are established between the corresponding clusters at

Table 4: Number of ports used at each layer: JPN25 model

Layer	Number of ports
0th layer	140
1st layer	70

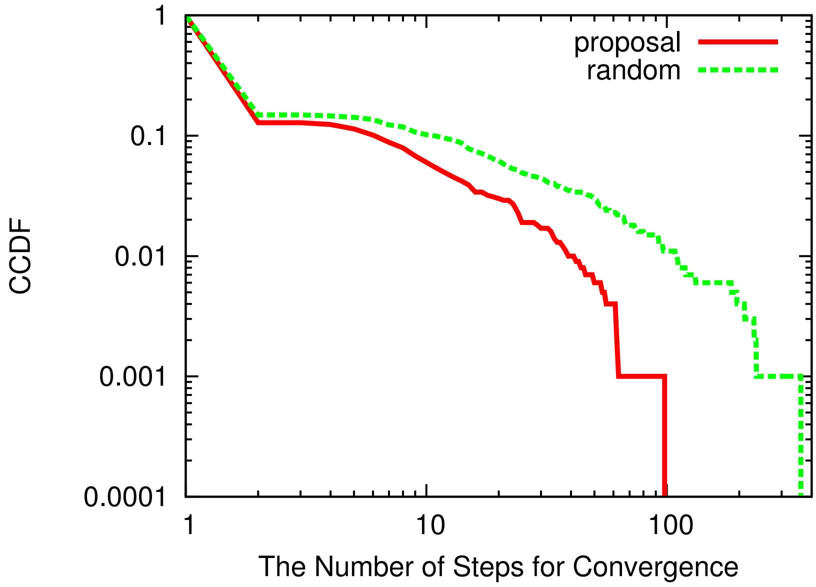

Figure 18: Distribution of the number of steps for convergence: JPN25 model, 10-ports

the bottom layer. In this way, we connect seven VNT candidates at the top layer and one VNT candidate at the bottom layer. Finally, we obtain seven VNT candidates.

Table 4 shows the number of ports used at each layer in the VNT candidates. We can see that the number of ports at the top layer is twice of that of the bottom layer.

Fig. 18 shows distribution of the number of steps for convergence. Traffic demands and a method for comparison are similar to those described in Section 5.2. The horizontal axis shows the number of steps for convergence and the vertical axis shows CCDF of the number of steps. We can find that our VNT control that uses the VNT candidates by our method as attractors needs less steps for convergence. Since our VNT control method using the VNT candidates by our method as attractors finds a solution, i.e., a VNT that can accommodate traffic, within a shorter time, our method can design better VNT candidates than the other method even when we take the physical

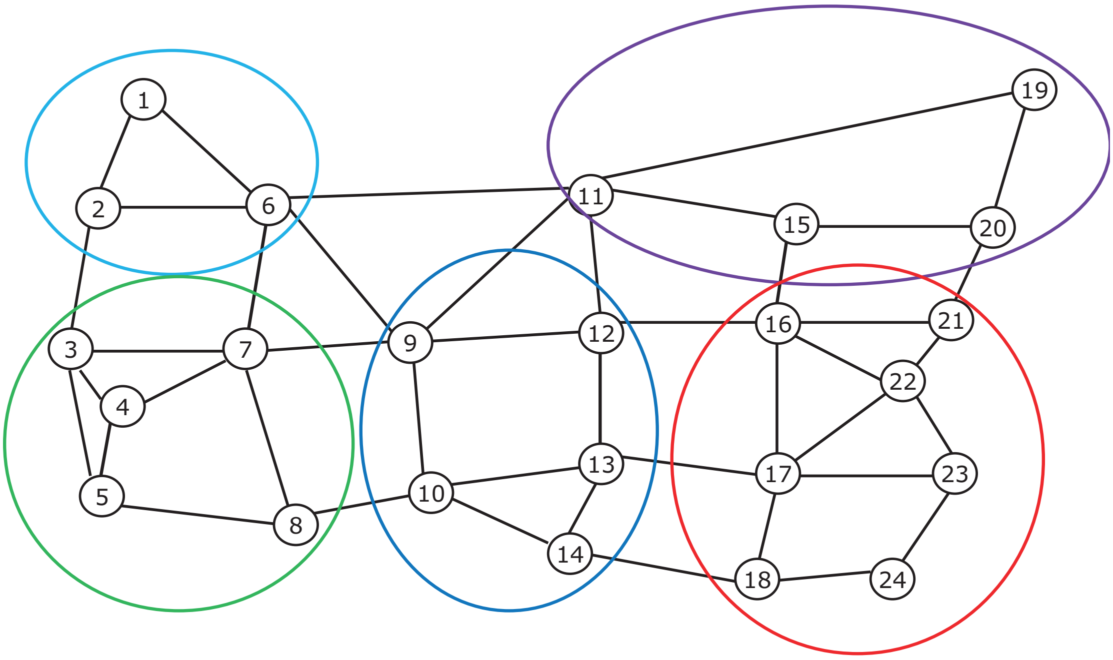

Figure 19: Clusters in USNET

network topology into account.

5.3.2 Evaluation for USNET

USNET has 24 nodes and each node has 10 ports. We divide the physical network into five clusters, as shown in Fig. 19. The nodes surrounded by a circle belong to the same cluster: one cluster has seven nodes, two clusters has five nodes, one cluster has four nodes and one cluster has three nodes. We consider USNET as the two-layer network, which consists of the top layer whose nodes are the clusters and the bottom layer that is equal to the original network. A topology at the top layer is regarded to be composed of five nodes with three router ports and we obtain seven VNT candidates by following Step.3 of our structural reduction method. A VNT candidate in each cluster at the bottom layer has a full-mesh topology. When a lightpath is established between two nodes at the top layer, four bidirectional lightpaths are established between the corresponding clusters at the bottom layer. In this way, we connect seven VNT candidates at the top layer and one VNT candidate at the bottom layer. Finally, we obtain seven VNT candidates.

Table 5 shows the number of ports used at each layer in the VNT candidates. We can see that

Table 5: Number of ports used at each layer: USNET

Layer	Number of ports
0th layer	102
1st layer	100

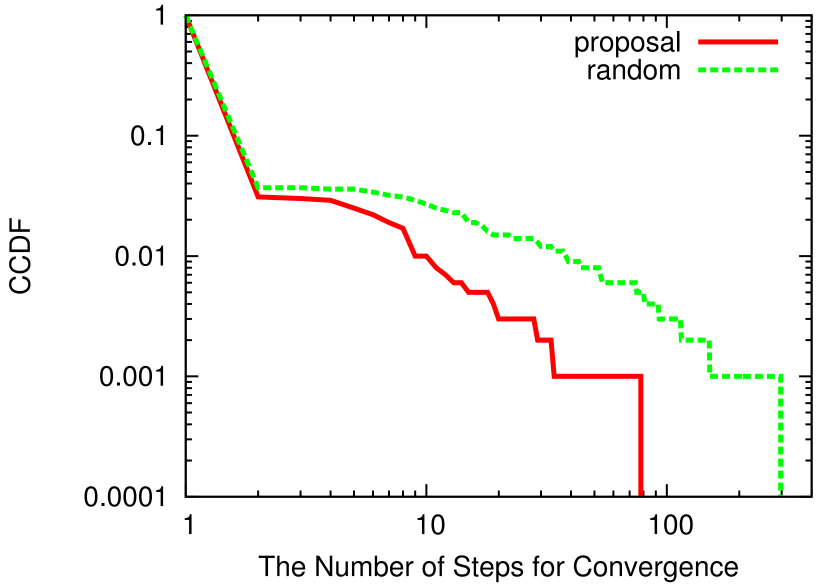

Figure 20: Distribution of the number of steps for convergence: USNET, 10-ports

the number of ports at the top layer is about the same as that of the bottom layer.

Fig. 20 shows distribution of the number of steps for convergence. Traffic demands and a method for comparison are similar to those described in Section 5.2. The horizontal axis shows the number of steps for convergence and the vertical axis shows CCDF of the number of steps. We can find that our VNT control that uses the VNT candidates by our method as attractors needs less steps for convergence. That is, our VNT control method using the VNT candidates by our method as attractors finds a solution within a shorter time. Our method can design better VNT candidates than the other method for another physical network.

As mentioned in Section 5.1, using more ports at an upper layer than a lower layer leads to suppression of the maximum link utilization. However, the number of ports used at the top layer is about the same as that of the bottom layer in USNET, while the number of ports used at the top layer is twice of that of the bottom layer in JPN25 model. The number of clusters is the same in both networks. Therefore, difference in cluster size, i.e., the number of nodes in a cluster, causes the difference in the number of ports used at each layer. The cluster size in USNET varies widely since the cluster size in USNET is three to seven, while the cluster size in JPN25 model is four to six. When the cluster size is large, a lightpath inside the cluster can be a long-distance link. That is, establishing more lightpaths inside clusters contributes to reduction of the average hop length. In general, the smaller the average hop length is, the smaller the maximum link utilization on a VNT is. Increasing lightpaths inside clusters, i.e., the number of ports used at the bottom layer, results in assigning about the same number of ports at each layer. Therefore, it is necessary to adjust the number of ports used at each layer depending on a physical network topology, i.e., cluster size and the number of clusters. We can adjust the number of ports used at each layer by changing a topology in each cluster at the bottom layer, such as a full-mesh topology or a star topology with several hub nodes. When we change a topology in each cluster at bottom layer from a full-mesh to a star, we can use the remaining ports to establish lightpaths between clusters. Actually, we use a star topology with two hub nodes in each cluster at the bottom layer for JPN25 model to establish more lightpaths between clusters. And then, we use a full-mesh topology for USNET to establish more lightpaths inside clusters.

6 Conclusion

In this paper, we proposed a method to design attractors for our VNT control method based on attractor selection. Our basic approach prepared a limited number of attractors with diversity by classifying VNT candidates into groups on the basis of their characteristics and selecting an attractor from each group. In order to design attractors for large-scaled networks, we also proposed a structural reduction method that hierarchically contracts a network topology so that we can apply our approach to large-scaled networks. We showed that the VNT candidates obtained by our method can accommodate various traffic demands, so that our VNT control method can find a solution, i.e., a VNT that can accommodate IP traffic, within a shorter time guided by the attractors.

In our structural reduction method in Section 4, the number of ports used at each layer, i.e, the number of lightpaths established at each layer, is an important factor. From the evaluation results, we should basically use more ports at an upper layer in order to design better VNT candidates. This is because that establishing more lightpaths at an upper layer leads to reduction of traffic loads on lightpaths. However, the number of traffic patterns against which VNT candidates by our method can suppress the maximum link utilization than the method for comparison varies depending on networks. Therefore, we investigate appropriate number of ports at each layer depending on the number of ports each node has, the number of clusters and the cluster size.

Acknowledgements

I owe my deepest gratitude to my supervisor, Professor Masayuki Murata of Osaka University, for his insightful comments and constructive feedback. I especially would like to show my greatest appreciation to Associate Professor Shin'ichi Arakawa of Osaka University. I obtained detailed advice from face-to-face discussion about my work with him. I also would link to offer my special thanks to Associate Professor Yuki Koizumi, for his valuable and useful suggestions . Without their persistent supports, this thesis would not have been possible. I am deeply grateful to Associate Professor Yuichi Ohsita and Associate Professor Daichi Kominami, for helpful comments. Finally, I would like to thank all the members of Advanced Network Architecture Research Laboratory of Osaka University, for their encouragement and supports.

References

[1] J. Wu, "A survey of WDM network reconfiguration: Strategies and triggering methods," *Computer Networks*, vol. 55, pp. 2622–2645, May 2011.

[2] B. Ramamurthy, R. K. Sinha, and K. Ramakrishnan, "Multi-layer design of IP over WDM backbone networks: Impact on cost and survivability," in *Proceedings of Design of Reliable Communication Networks*, pp. 60–70, Mar. 2013.

[3] D.-R. Din and C.-W. Chou, "Virtual-topology adaptation for mixed-line-rate optical WDM networks under dynamic traffic," in *Proceedings of International Conference on Computer Communications and Networks*, pp. 1–6, Aug. 2014.

[4] X. Zhang, H. Wang, and Z. Zhang, "Survivable green IP over WDM networks against double-link failures," *Computer Networks*, vol. 59, pp. 62–76, Feb. 2014.

[5] B. Mukherjee, D. Banerjee, S. Ramamurthy, and A. Mukherjee, "Some principles for designing a wide-area WDM optical network," *IEEE/ACM Transactions on Networking*, vol. 4, pp. 684–696, Oct. 1996.

[6] R. Ramaswami and K. Sivarajan, "Design of logical topologies for wavelength-routed optical networks," *IEEE Journal on Selected Areas in Communications*, vol. 14, pp. 840–851, June 1996.

[7] F. Ricciato, S. Salsano, A. Belmonte, and M. Listanti, "Off-line configuration of a MPLS over WDM network under time-varying offered traffic," in *Proceedings of IEEE INFOCOM*, vol. 1, pp. 57–65, June 2002.

[8] G. Agrawal and D. Medhi, "Lightpath topology configuration for wavelength-routed IP/MPLS networks for time-dependent traffic," in *Proceedings of IEEE GLOBECOM*, Nov. 2006.

[9] B. Ramamurthy and A. Ramakrishnan, "Virtual topology reconfiguration of wavelength-routed optical WDM networks," in *Proceedings of IEEE GLOBECOM*, vol. 2, pp. 1269–1275, Nov. 2000.

[10] S. Gieselman, N. Singhal, and B. Mukherjee, "Minimum-cost virtual-topology adaptation for optical WDM mesh networks," in *Proceedings of International Conference on Communications*, vol. 3, pp. 1787–1791, June 2005.

[11] N. Sreenath and C. S. R. Murthy, "On-line reconfiguration of virtual topologies in wavelength-routed WDM networks," *Journal of High Speed Networks*, vol. 12, pp. 141–169, Jan. 2002.

[12] W. Yao and B. Ramamurthy, "Rerouting schemes for dynamic traffic grooming in optical WDM mesh networks," in *Proceedings of IEEE GLOBECOM*, vol. 3, pp. 1793–1797, Nov. 2004.

[13] A. Gençata and B. Mukherjee, "Virtual-topology adaptation for WDM mesh networks under dynamic traffic," *IEEE/ACM Transactions on Networking*, vol. 11, pp. 236–247, Apr. 2003.

[14] Y. Ohsita, T. Miyamura, S. Arakawa, S. Ata, E. Oki, K. Shiomoto, and M. Murata, "Gradually reconfiguring virtual network topologies based on estimated traffic matrices," *IEEE/ACM Transactions on Networking*, vol. 18, pp. 177–189, Feb. 2010.

[15] I. Ari, B. Hong, E. L. Miller, S. A. Brandt, and D. D. Long, "Managing flash crowds on the Internet," in *Proceedings of Modeling Analysis and Simulation of Computer and Telecommunication Systems*, pp. 246–249, Oct. 2003.

[16] B. Li, G. Y. Keung, S. Xie, F. Liu, Y. Sun, and H. Yin, "An empirical study of flash crowd dynamics in a P2P-based live video streaming system," in *Proceedings of IEEE GLOBECOM*, pp. 1–5, Nov. 2008.

[17] A. Schaeffer-Filho, P. Smith, A. Mauthe, and D. Hutchison, "Network resilience with reusable management patterns," *IEEE Communications Magazine*, vol. 52, pp. 105–115, July 2014.

[18] Y. Koizumi, T. Miyamura, S. Arakawa, E. Oki, K. Shiomoto, and M. Murata, "Adaptive virtual network topology control based on attractor selection," *IEEE/OSA Journal of Lightwave Technology*, vol. 28, pp. 1720–1731, June 2010.

[19] Y. Minami, S. Arakawa, Y. Koizumi, T. Miyamura, K. Shiomoto, and M. Murata, "Adaptive virtual network topology control in WDM-based optical networks," in *Proceedings of INTERNET*, pp. 49–54, Sept. 2010.

[20] Y. Baram, "Orthogonal patterns in binary neural networks," *NASA Technical Memorandum No. 100060*, Mar. 1988.

[21] C. Furusawa and K. Kaneko, "A generic mechanism for adaptive growth rate regulation," *PLoS Computational Biology*, vol. 4, p. e3, Jan. 2008.

[22] R. Rojas, *Neural networks: a systematic introduction*. Springer, July 1996.

[23] D. Banerjee and B. Mukherjee, "Wavelength-routed optical networks: Linear formulation, resource budgeting tradeoffs, and a reconfiguration study," *IEEE/ACM Transactions on Networking (TON)*, vol. 8, pp. 598–607, Oct. 2000.

[24] V. D. Blondel, J.-L. Guillaume, R. Lambiotte, and E. Lefebvre, "Fast unfolding of communities in large networks," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2008, Oct. 2008.