

Master's Thesis

Title

**Assortativity's effect on efficiency and robustness in inter-connected
networks**

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Abstract

Information networks have been growing rapidly and increasing in complication. As the scale of a network increases, the computational and communication cost for controlling and managing the network drastically increases. Since information networks are and will be the significantly important infrastructure, they should also have the high reliability and efficiency as well as the scalability in the control and the management. A inter-connected network is a network that is composed of a number of networks. An example of the network of networks is the Internet. In most cases, a network that is a component of a network of networks is not too large to be controlled and managed. Since no administrator manages the whole of the future network, the reliability and the efficiency of the network will not be guaranteed. Therefore, we examine how to design the inter-connected network for more robustness and efficiency. As a clue to the design of inter-connected networks, we focus on the human-brain network which has similar structure to the inter-connection of networks. The human-brain can handle a wide variety of tasks adaptively. In the human brain, anatomical connections make a characteristic network that has high topological efficiency and robustness while minimizing wiring cost. These advantages have been obtained in the process of brain growth and evolution. The human-brain network has some topological features as seen in complex networks, such as modular structure and assortative mixing. Because the assortativity of a network is known to have a relation to the robustness, we examine the assortativity of inter-connected networks. In this thesis, we clarify the effect of the assortativity on the robustness and the efficiency in an inter-connected network. For that purpose, we construct a network that has a desired assortativity and also construct a inter-connected network where the assortativity between component networks has a desired value. On the assortativity of a single network, we reveal that an increase in the assortativity of a network causes an increase in the hop count, high robustness of connectivity against the failure of high-degree nodes, a low information diffusion efficiency, and a

concentration of communication loads on a few edges. On the assortativity between networks, we reveal the following: Assortative connections between networks shortens the average hop length and enhances robustness, disassortative connections between networks distributes communication loads of inter-network links, and disassortative connection between networks is weak against the failures of high-degree nodes. Finally, we discuss the application of our results for the design of robust information networks.

Keywords

Inter-connected Network

Assortativity

Brain networks

Modular structure

Graph theory

Internet of Things

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1 Introduction

Information networks have been growing rapidly and increasing in complication. A lot of sensor devices which collect a variety of environmental information will be placed all over the places and connected to the Internet [1, 2]. The number of such devices connected to the Internet will be tens of billions in 2020 [3]. In addition, the services running over the network for human life will further diversify and the network will change to meet the various requirements. Then, the control and the management of huge networks such as Internet of Things (IoT) will be very hard to be dealt with because of their increases in communication cost and computational cost. Information networks are and will be the significantly important infrastructure, hence they should have the high reliability and efficiency as well as the scalability in the control and the management.

For these properties of information networks, we focus on the inter-connecting structure of multiple networks. A number of networks whose scale is not too large to be controlled and managed constitute a large network called inter-connected network. As is well known, the Internet is one of the inter-connected networks where numerous networks operated by Internet service providers are mutually connected [4]. In future information networks, an enormous number of networks, which are managed by different administrators, including the Internet will be inter-connected [5]. Since there will be no administrator who manages the whole of the network, the reliability and the efficiency of the network will not be guaranteed. Therefore, we examine how to design the inter-connected network for more robustness and efficiency.

Similar structures to the inter-connection of networks are seen in regulatory gene networks, protein-protein interaction networks, and human brain networks, which are called modular structures [6]. In a modular structure, a module is defined as a subset of network units so that connections between them are denser than connections with the rest of the network. Recently, due to the advances in neuroimaging techniques, the human brain can be analyzed by much finer spatial resolution. Then, a structural network of the brain represented by the anatomical connections among the region of interest has been studied. Brain networks are found to have high topological efficiency and robustness while minimizing wiring cost. Furthermore, the human brain can tackle a large variety of tasks adaptively. These advantages are considered to have been obtained in the process of human growth and evolution.

As a clue to the design of inter-connected networks, we pay attention to the human brain net-

work. The human brain is a complicated network composed of neuronal cell bodies residing in cortical grey matter regions, joined by axons, protected by myelin. Advances in an analytical method has revealed that the brain network has the topological features seen in complex networks such as small-world properties, hierarchical modular structure and assortative structure [7–9]. These topological features are considered to give advantages to the brain such as robustness against node failures and efficiency to tackle tasks adaptively [10]. Application to the information network is being actively discussed [11, 12].

In order to apply the structural properties of the human brain, it is essential to clarify the effects of the structural properties of the human brain. Small-world properties of brain networks are known to lead to their communication efficiency. However, their topological advantages brought by the hierarchical modularity are unrevealed. Opinions are divided: it brings about communication efficiency, robustness, maintaining dynamical activity, and adaptive evolution. It is significant to reveal how these topological properties contribute to the function of brain networks for understanding the human brain and for its technological application.

In this thesis, taking the modular structure into account, we focus on the distinctive degree correlation of brain networks, called assortativity. Assortativity represents the degree correlations between connected nodes. If a network shows high assortativity (assortative mixing), nodes with similar degree tend to be connected each other. On the other hand, in a network with low assortativity (disassortative mixing), nodes that differ much from their degree are preferentially connected each other [13]. Generally, an assortative-mixing network is robust against selective node failure and accelerates the spreading of information generated from high-degree nodes [14, 15]. Brain networks show modular structure, where densely connected groups of nodes construct modules and they are sparsely connected with each other, and each module presents assortative mixing. However, in previous research, the effect of the degree correlation of edges between modules has not been studied.

We clarify the effect of the assortativity in inter-connected networks on robustness and efficiency. Furthermore, the interinfluence between the assortativity within a network and the assortativity between networks should be explored in detail. For this purpose, we make networks that have different assortativity and analyze them through following metrics: (1) the average hop length, (2) robustness against selective node failure, in which the highly connected nodes are selectively removed, (3) the edge betweenness centrality [13], and (4) each node's importance on

information diffusion [16, 17]. First, we focus on a single network and reveal the basic properties of an assortative network. Then, we target at an inter-connected network.

The rest of this thesis is organized as follows. Section 2 shows the definition of assortativity and the method for revealing the effect of assortativity on the robustness and the efficiency of inter-connected networks. Simulation results are shown in Section 3. In Section 4, we summarize the results and discuss the application of our results for designing the information network. In Section 5, we conclude this thesis and describe future work.

2 Method

2.1 Overview

In this section, we mention an overview of our method for revealing the effects of assortativity. We focus on two types of assortativity: within a network and between networks. To examine the influence on the robustness and the efficiency of two types of assortativity, we propose network construction methods for a desired assortativity. At first, we show a construction method for a single network that has a specific assortativity and then, show a construction method for an inter-connected network, which consists of two networks constructed by the above method, so that edges connecting the networks with a specific assortativity. An example of an inter-connected network is shown in Fig. 1.

To begin with, we use a single network and reveal the properties of the assortativity within a network. Then, we target at an inter-connected network and reveal the properties of the assortativity between networks. Moreover, we examine the interaction between the assortativity within a network and the assortativity between networks. To examine the influence of assortativity, we make networks that have different assortativity and analyze them through some metrics from the point of view of graph theory.

In Subsection 2.2, we explain the definitions of assortativity within a network and assortativity between networks. In Subsection 2.3, we show the method to construct a network with different assortativity. Although assortativity could have an influence on various metrics, taking essential requirements for information networks into consideration, we use four metrics for our evaluation: (1) the average hop length, (2) robustness against selective node failure, (3) the edge betweenness centrality, and (4) each node's importance on information diffusion. In Subsection 2.4, we mention the metrics in details.

2.2 Definition of Assortativity

In this subsection, we explain the definition of assortativity. We define the assortativity within a network and between networks. The assortativity within a network is measured by the *assortativity coefficient* proposed by Newman [18]. The assortativity coefficient is shown in Subsection 2.2.1. On the other hand, measuring method for the assortativity between networks has been not defined to our knowledge. Therefore, we exploit the *universal assortativity coefficient* [19] which is

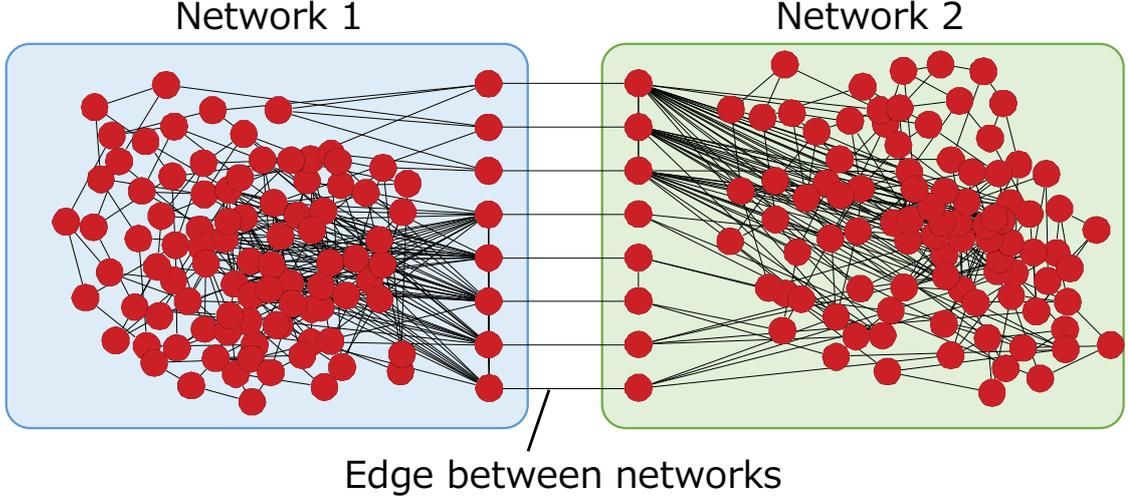


Figure 1: Inter-connected network

the dilatation of the assortativity coefficient. The universal assortativity coefficient is mentioned in Subsection 2.2.2.

2.2.1 Assortativity within a Network

The assortativity of a network is proposed by Newman as the assortativity coefficient [13]. The assortativity coefficient is calculated from the remaining degree distribution $q(k)$ defined by

$$q(k) = \frac{(k+1)p(k+1)}{\sum_j jp(j)}, \quad (1)$$

The remaining degree distribution is related to the degree distribution $p(k)$, which describes the probability that the degree of a randomly chosen node is k . The remaining degree means the number of edges leaving the vertex other than the one we arrived along. That is, this is less than node's degree by one. Given $q(k)$, joint probability distribution $q(j, k)$ can be introduced, which means the probability that two endpoints of a randomly chosen edge have remaining degree k and j . Then, assortativity coefficient r is defined as following

$$r = \frac{1}{\sigma_q^2} \left[\sum_{j,k} jkq(j, k) - \left(\sum_j jq(j) \right)^2 \right], \quad (2)$$

where σ_q is the standard deviation of the remaining degree distribution $q(k)$ defined by $\sigma_q^2 = \sum_k k^2q(k) - \left(\sum_j jq(j) \right)^2$. The range of values that r can take is $[-1, 1]$. It takes positive value

when the network is assortative. On the contrary, it takes negative value when the network is disassortative. And if it takes zero or near zero value, nodes are randomly connected with each other regardless of their degrees. The range of value r can take depends on the number of nodes, degree distribution.

2.2.2 Assortivity between Networks

The universal assortativity coefficient is proposed in [19]. We use it to define the assortativity between networks. This coefficient reflects an individual edge's contribution to the global assortativity coefficient of the whole network. The authors of [19] utilize it to analyze the assortativity of any part of a network. The universal assortativity coefficient for a set of targeted edges E_{target} is represented by the sum of each edge's contribution to the assortativity of the whole network, which is described in Subsection 2.2.1. Each edge's contribution to the global assortativity is derived from global assortativity r in 2. Global assortativity r can be rewritten by:

$$\begin{aligned}
r &= \frac{1}{\sigma_q^2} \left[\sum_{j,k} jkq(j,k) - \left(\sum_j jq(j) \right)^2 \right] \\
&= \frac{1}{\sigma_q^2} \left[\sigma_{jk} jkq(j,k) - U_q^2 \right] \\
&= \frac{1}{\sigma_q^2} \left[\sigma_{jk} jkq(j,k) - \sum_j jq(j) - \sum_k kq(k) + U_q^2 \right] \\
&= \frac{1}{\sigma_q^2} \left[\sigma_{jk} jkq(j,k) - \sum_j \sum_k kq(j,k) - \sum_k kq(k) + U_q^2 \right] \\
&= \frac{1}{\sigma_q^2} \left[\sum_{jk} jkq(j,k) - \sum_j \sum_k (j+k)q(j,k) + U_q^2 \right] \\
&= \frac{1}{\sigma_q^2} \left[\sum_{jk} (j - U_q)(k - U_q)q(j,k) \right] \\
&= \frac{1}{\sigma_q^2} \left[E(J - U_q)(K - U_q) \right],
\end{aligned}$$

where $U_q = \sum_j jq(j)$ is the expected value of remaining degree, J and K are variables of the remaining degree, which have the same expected value U_q . Then, the contribution ρ_e of edge e is defined by:

$$\rho_e = \frac{(J - U_q)(K - U_q)}{M\sigma_q^2}, \quad (3)$$

where M is the number of edges in the whole network, and j, k are the remaining degrees of the two endpoints of edge e . Finally, the universal assortativity coefficient ρ is defined as:

$$\rho = \sum_{e \in E_{target}} \rho_e = \sum_{e \in E_{target}} \frac{(J - U_q)(K - U_q)}{M\sigma_q^2}. \quad (4)$$

The universal assortativity coefficient ρ is a part of global assortativity. So, if we set E_{target} to the set of the whole edges E , ρ is equal to the Newman's global assortativity. In this thesis, all edges between networks are defined as an element of E_{target} . And the assortativity between networks is calculated by Eq.(4) with this E_{target} .

2.3 Network Construction Methods for Different Assortativity

In this subsection, we propose two methods for constructing a network. In Subsection 2.3.1, we propose a method for constructing single network that has the specific assortativity within the network. In Subsection 2.3.2, we propose the other method for constructing an inter-connected network whose assortativity between two component networks has the specific values.

2.3.1 Single Network with Different Assortativity within a Network

We construct a network which has target assortativity by repeatedly rewiring edges of a given network. In this method, we changes assortativity without changing the degree distribution because each rewiring does not change degree distribution.

A repeated rewiring method of edges in the network is in the following way. First, we randomly choose two edges so that the edges do not share their endpoints. Then, four nodes are selected as the endpoints of edges. Two pairs of nodes are rewired so that r approaches the desired value as shown in Fig. 2. In the rewiring process, it is determined which nodes are connected with each other only by their degree. To raise assortativity, two nodes whose degrees is higher than the others are wired. On the contrary, the assortativity goes down in the other rewiring methods. The number of the other rewiring pattern is two. We select more effective pattern for lowering assortativity by calculating both of the assortativity coefficient. Note that selected two edges are not rewired if the original connection pattern is most suitable.

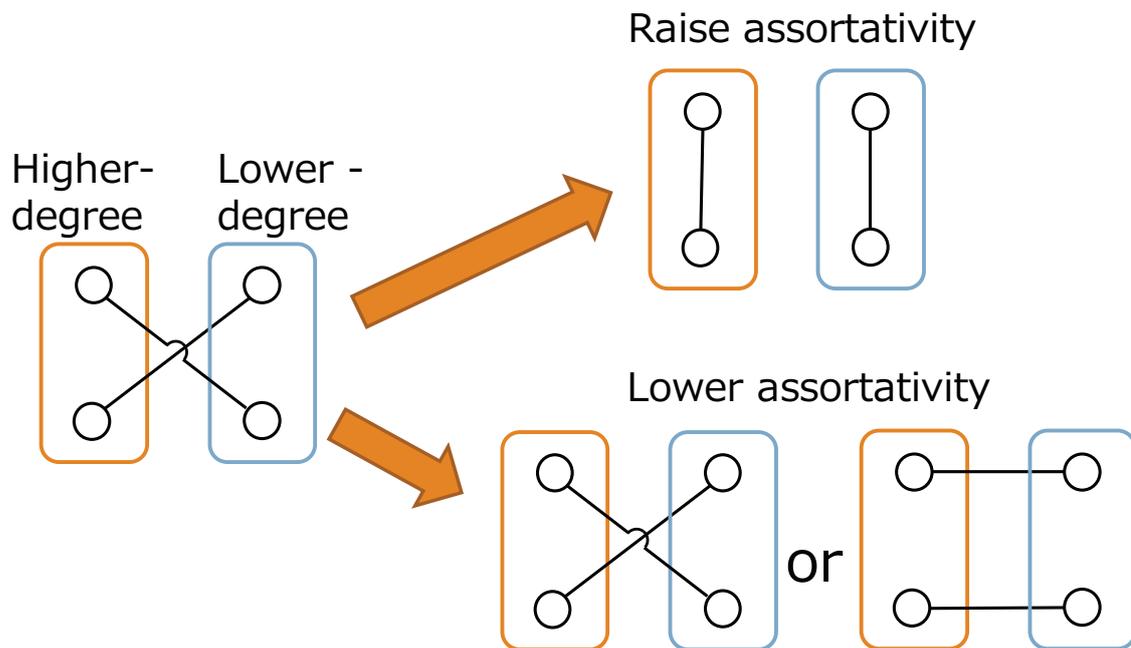


Figure 2: Rewiring pattern

2.3.2 Inter-Connected Network with Different Assortativity between Networks

To construct an inter-connected network which has the specific assortativity between networks, two same networks are connected by M edges. To decide a suitable mixing pattern, we repeatedly delete and add an edge between networks. This is done by stochastically. The mixing pattern is determined as following.

1. We randomly connect two networks via M edges.
2. The assortativity between networks is calculated. If the assortativity between networks is within an acceptable range for the target value, the connecting pattern at this point is adopted. Otherwise, the following process is repeated until the assortativity between networks reaches the value within the range.
3. One edge between networks is randomly chosen and deleted.
4. One edge between networks is randomly added. If the assortativity does not approach the target value, we delete the added edge and re-add the edges deleted at 3. Then, redo the selection of additional link.

2.4 Metrics for Evaluation

Metrics for our evaluation are (1) the average hop length, (2) robustness against selective node failure, (3) the edge betweenness centrality, and (4) each node's importance on information diffusion. We describe the details for them in the following subsections.

Average Hop Length The average hop length is the average hop count of a shortest-hop path from any node to any node, which is used widely in the field of graph theory. This metric presents data transfer efficiency. A network with a shorter average hop length is expected to achieve a high speed data transfer.

Robustness Robustness is evaluated by using the giant component size when some nodes are removed. Giant component size is the number of nodes in the largest connected component. The network that maintains the giant component size is regarded to have higher robustness. Note that a removed node is selected from the highest degree nodes in the remaining network. If different nodes have the same degree, one node is randomly selected from them. This removal model can be used as the model of targeted attacks, power depletion of a load-concentrated nodes in wireless communication in terms of information networks.

Edge betweenness centrality The edge betweenness centrality is defined as the number of the shortest paths that go through an edge in a network. It can be considered as communication loads on edges. This metric shows the concentration of communication load in a network. In the context of the information network, if there are edges with high edge betweenness centrality, traffic congestion could be occur around these points.

each node's importance on information diffusion We use the susceptible-infected-recovered (SIR) model [20] to model the diffusion of information. In this model, each node is either susceptible (S), infected (I), or recovered (R). An infected node passes diseases to neighbor nodes with probability β , and recovers itself with probability γ . Consequently, all infected nodes will be recovered. This means that the network is composed of susceptible nodes and/or recovered nodes at the end. The rate of recovered nodes in the network shows the scale of epidemic, which can be interpreted as the scale of information diffusion in the field of information networks. By setting each node as

the infected node at the initial step, we use this scale (\mathcal{E}) as each node's importance on information diffusion.

Although many other models are possible to model the diffusion of information, in this thesis, we use SIR model for two reasons: First, it is used widely in modeling the diffusion of information. Second, because of its immediate convergence.

3 Results

3.1 Single Network

In this section, we investigate a single network with various assortativity. We use a scale-free network (SF network) that has 100 nodes and 295 edges, and each node has at least 3 edges. This network is generated by the BA model [21]. The initial value of assortativity r of this network is -0.12 . By rewiring edges of this network as explained in Subsection 2.3.1, we generate networks with different assortativity. When we try to rewire the edges of this network, assortativity varies within a range of $-0.67 \leq r \leq 0.58$.

3.1.1 Average Hop Length

The relation between the average hop length and assortativity is shown in Fig. 3. Fig. 3 shows that as r becomes larger, the average hop length increases. Specifically, the average hop length rapidly increases when $r \geq 0.5$. In information networks, an increase in the average hop length often leads to performance degradation such as an increase in the communication delay.

To clarify the reason why the average hop length increases as assortativity becomes larger, we show the topology whose assortativity is the highest ($r = 0.58$) in Fig. 4. In this topology, almost all of nodes are connected with the same-degree nodes and a set of nodes with the same degree form a cluster. And these clusters are concatenated in order of degree. This tendency did not change even if the degree distribution has changed. Generally, a chain-like topology has a longer average hop length (unlike the small-world topology), and therefore, a highly assortative topology has a longer average hop length. This tendency was to be much strong in the case of increasing assortativity.

Why does the rapidly increases of average hop length happen when $r \geq 0.5$? The main reason of this is the loss of shortcut edges. In [22], it is said that a few of shortcut edges can drastically shorten the average hop length. When $r \geq 0.5$, this shortcut edges are likely to be lost because in clustered topology as shown in Fig. 4. When r is too high, a network lose these edges. Consequently, the average hop length rapidly increases when $r \geq 0.5$.

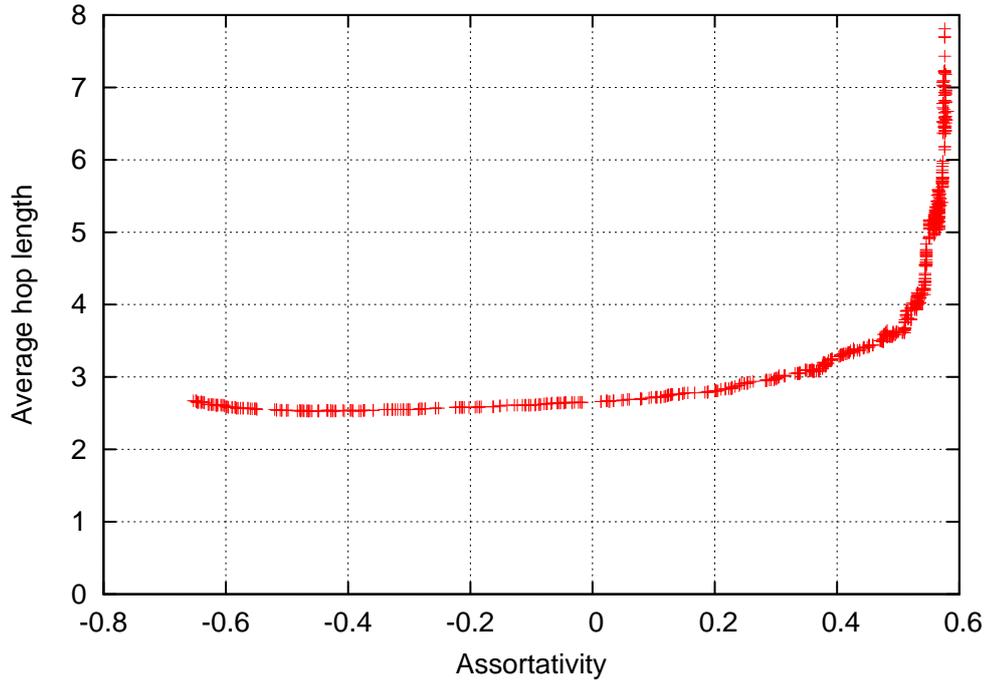


Figure 3: Average hop length in a single network

3.1.2 Robustness

Figure 5 shows the giant component size of networks with different assortativity when nodes are removed in descending order of degree. When $r < 0.5$, the giant component size decreases as the assortativity becomes high. A decrease tendency of giant component size depends on the assortativity. As stated in Subsection 3.1.1, an assortative topology consists of some clusters that are connected with each other like a chain. In our selective node-failure scenario, node failure occurs from the high-degree side of this chain. Therefore, nodes which have a lower degree remain to be connected with each other even if high degree nodes fail. This is because an assortative topology is robust against selective node failure.

Meanwhile, a topology with extremely high assortativity (the topology with $r = 0.58$) shows lower robustness than these with lower assortativity. This topology has a smaller number of edges between clusters because edges between clusters connects nodes whose degree are different each other. Therefore, connectivity between clusters is easily lost in excessively high assortative topology. In other words, a topology with extremely higher assortativity is sensitive to selective node

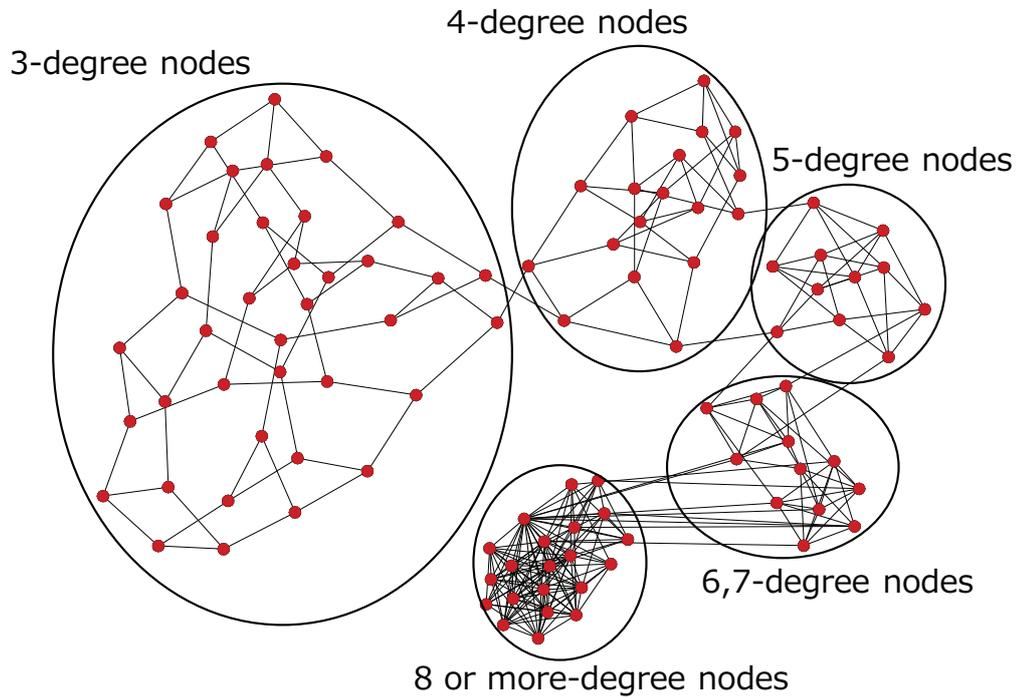


Figure 4: Topology with $r = 0.58$

failure.

3.1.3 Information-Diffusion Efficiency

Each node's importance on information diffusion is shown in Fig. 6. In the figure, x-axis means nodes arranged in the increasing order of \mathcal{E} . Simulations are executed 50 times for each node as the infected node at the initial step. The result for one node is the average of 50 times simulation. The value of β is set to 0.08. The value of γ is set to 0.10. Figure 6 shows that it is difficult to diffuse information when a topology has high assortativity. The hop length between nodes has great influence on the probability of transmitting information from one node to another node when using the SIR model for information diffusion. For that reason, it is clear that scale E of each node importance on information diffusion is higher in the network as r get higher.

To examine the influence of assortativity itself on the information diffusion, we need to compare information diffusion of two topologies which has the same average hop length but different assortativity. For this purpose, we construct two topologies by the following method. The

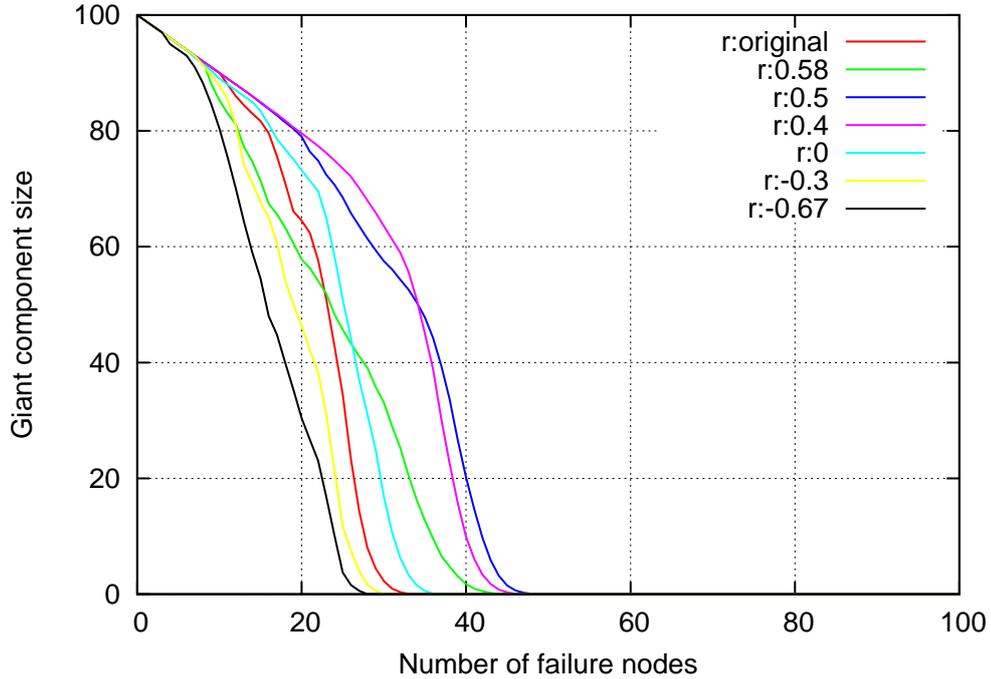


Figure 5: Giant component size in a single network

overview of this method is simple and similar to the method for constructing topology that has target assortativity explained in Subsection 2.3.1. First, we generate two topologies with $r = -0.67$ and $r = 0.3$. Then we repeatedly rewire edges of the topology with $r = -0.67$ so that average hop length increases. In this method, we select two edges randomly and rewire them. Only if the average hop length of the topology with $r = -0.67$ approaches that of the topology with $r = 0.3$ as the result of rewiring, this rewiring process is executed. Otherwise edge selection is done over again. By repeating the rewiring procedure, we finally obtain two topologies which have the same average hop length but different assortativity.

The average hop length of two topologies generated by the method is equally 3.05. Assortativity of these topologies are 0.3 and -0.54 . Each node's importance on information diffusion of the two topologies is shown in Fig. 7. Figure 7 shows that in the assortative topology, the spread of information is larger than the disassortative topology. In the SIR model, the probability that a node transmit information to other nodes depends on the number of its neighbor nodes because the number of nodes which can be infected at the next step equals the number of susceptible nodes neighboring an infected nodes. Therefore, low degree nodes have a low probability of diffusing

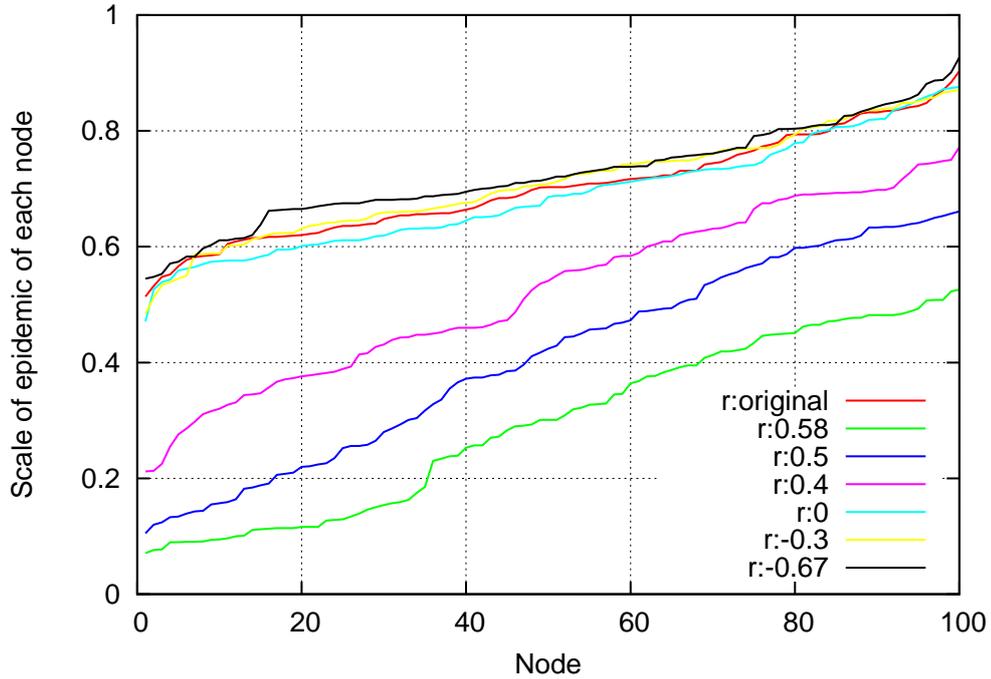


Figure 6: Information-diffusion efficiency in a single network

information. Furthermore, in an assortative network, low degree nodes have few connections to high degree nodes, which makes the network have low efficiency of information diffusion. Figure 8 shows the number of infections of each node in the topology with $r = 0.55$ and -0.3 . The total number of simulation for one topology is 5,000 times (50 simulations for each node). As for the topology with $r = -0.3$, all nodes are uniformly infected. On the contrary, as for the topology with $r = 0.55$, there are an area where information does not likely diffuse. In a highly assortative topology, nodes are not likely to be infected around this area.

3.1.4 Edge Betweenness Centrality

Figure 9 shows edge betweenness centrality of each edge in a single network. In the figure, X-axis represents edges arranged in the increasing order of edge betweenness centrality. From this figure, it is obvious that there are edges which shows extremely high edge betweenness centrality in the topology with $r = 0.58$. This is attributable to clustered structure as shown in Fig. 4. Since clusters are connected with each other via a few edges, many shortest paths go through these edges.

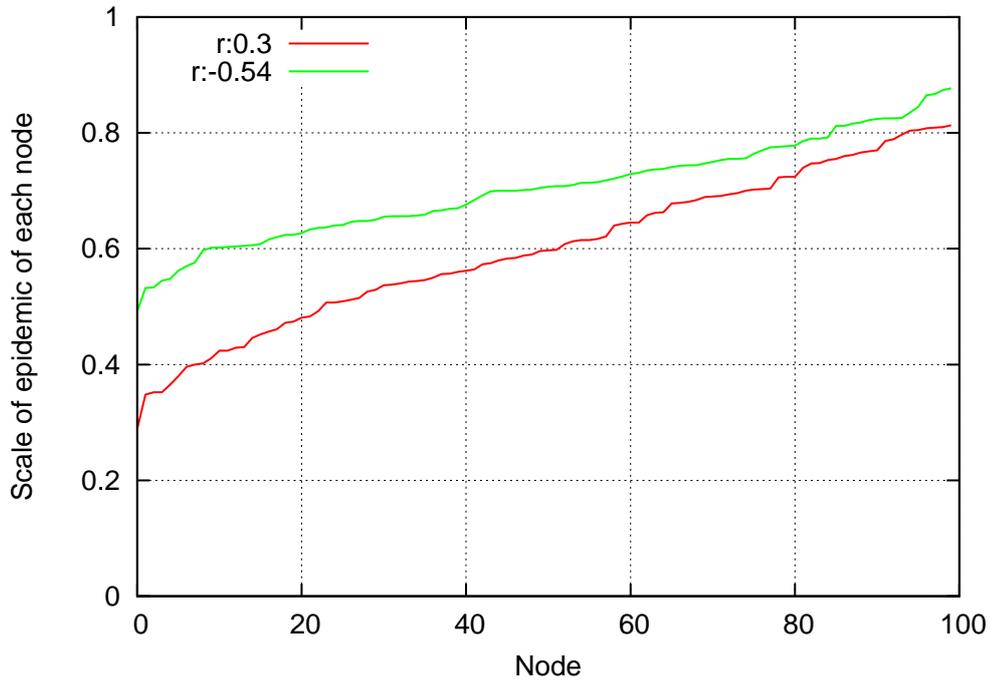


Figure 7: Information-diffusion efficiency in a single-module network

Table 1 shows the degrees of endpoints between which edges are in the top 10 edge betweenness centrality in the topology with $r = 0.58$. The table shows that edges connect clusters have high edge betweenness centrality.

Besides, as for the other topologies, edge betweenness centrality is focally high in assortative topologies. The reason for this is not equal to the case of the topology with $r = 0.58$. As described in Subsection 3.1.1, in an assortative topology, shortcut edges which connect separate-degree nodes shorten the average hop length. Because many shortest paths go through these edges, edge betweenness centrality is focally high. Table 2 shows the degrees of endpoints between which edges are in the top 10 edge betweenness centrality in topology with $r = 0.40$. The table shows that shortcut edges have high edge betweenness centrality.

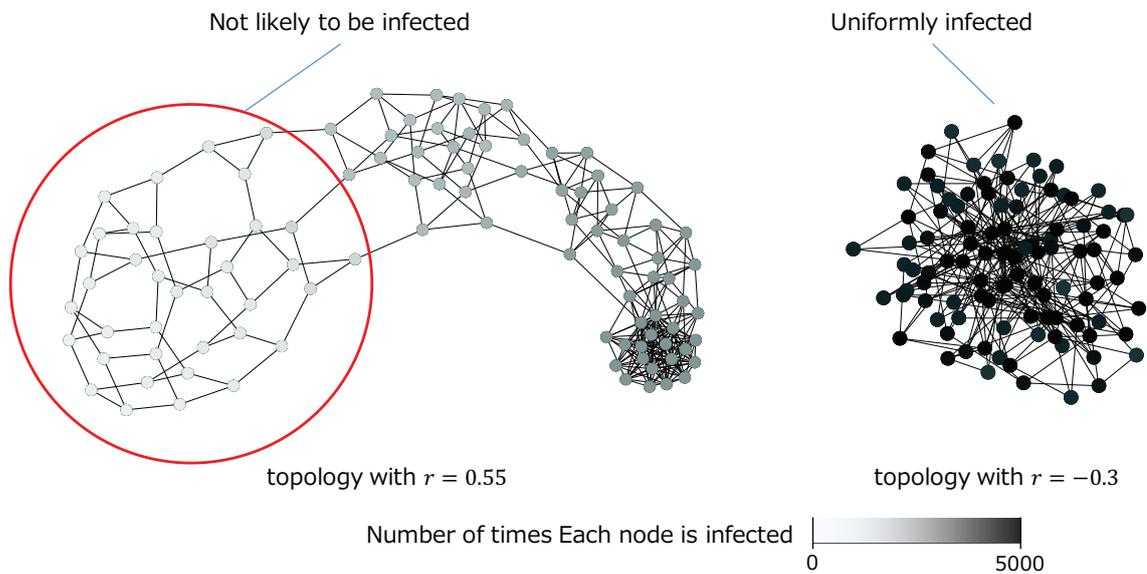


Figure 8: Number of infections of each node

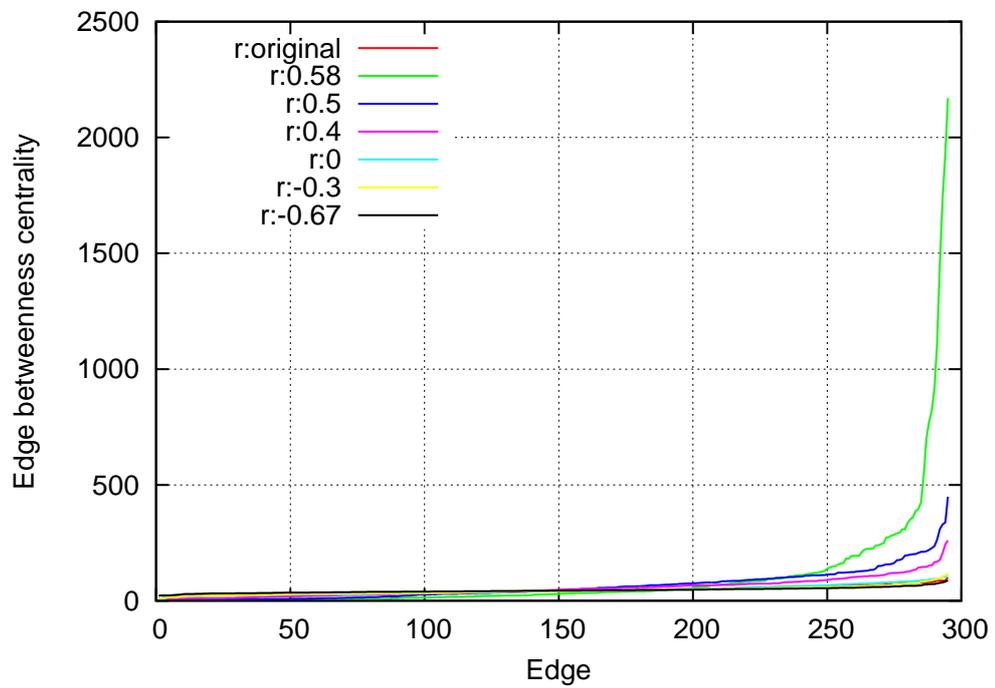


Figure 9: Edge betweenness centrality in a single network

endpoint 1	endpoint 2
5	4
6	5
4	5
4	3
6	5
6	7
7	6
4	4
4	4
5	4

Table 1: Degree of endpoints of the edges that have the top 10 edge-betweenness centrality ($r = 0.58$)

endpoint 1	endpoint 2
22	3
16	3
19	4
18	4
22	5
16	3
7	4
11	3
19	5
19	5

Table 2: Degree of endpoints of the edges that have the top 10 edge-betweenness centrality ($r = 0.40$)

3.2 Inter-Connected Network

In this subsection, we investigate the inter-connected network. We investigate inter-connected networks which have different assortativity between networks. An inter-connected network consists of the two same single networks. Each single network includes 100 nodes and 295 edges and two networks are connected with each other via 8 edges. To construct an inter-connected networks, we take two networks used for evaluation in Subsection 3.1. One is the assortative topology with $r = 0.3$. The other is the disassortative topology with $r = -0.3$. By using the method explained in Subsection 2.3.1, we construct an inter-connected network consisting of two assortative networks (AM-AM networks) and one consisting of two disassortative networks (DM-DM networks). Assortativity ρ between networks of an AM-AM topology and a DM-DM topology ranges from -0.0210 to 0.0304 . In our evaluation, we change targeted assortativity ρ for constructing AM-AM and DM-DM networks from -0.02 to 0.03 by 0.005 . Here, the acceptable error range for ρ is at most 0.001 . Because our construction method for an inter-connected network includes a probabilistic process, we make 10 topologies for each ρ and the results shown below are averaged values of 10 topologies.

3.2.1 Average Hop Length

Figures 10 and 11 show the average hop lengths in inter-connected networks. When we compare two topology, DM-DM topology shows shorter average hop length with each ρ . This is the result of the differences in average hop length within a single network. Comparing inter-connected networks with regard to different assortativity ρ , the average hop length decreases as ρ increases. This result is different from the one with a single network. Moreover, edges between networks have little effect on the overall structure of the network in comparison with edges within a network. Assortativity between networks does not have much influence on the average hop length than assortativity within a network. From this perspective, the assortativity within a network is completely different from the assortativity between networks.

Why does the average hop length decrease as the assortativity between networks increases? The main reason of this is connections between hub nodes (high-degree nodes). It is intuitive that the most effective edge connection between networks for shortening the average hop length of inter-connected networks is to link hub nodes because many shortest paths go through hub nodes

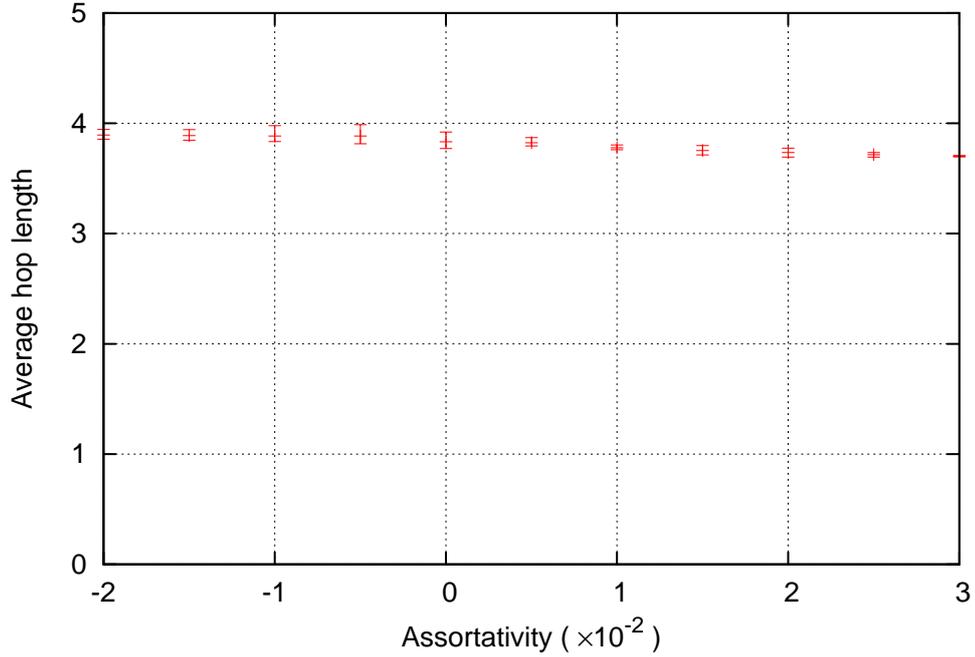


Figure 10: Average hop length in AM-AM networks

in the network. When we connect two networks assortatively, the inter-connected network has more edges connecting hub nodes than the network whose ρ is lower as shown in Tables 3 and 4.

This influence of hub nodes are found in the changes of closeness centrality. Closeness centrality measures how close a node is to all other nodes in a network. The node whose closeness centrality is the highest can be regarded as the most influential in terms of average hop length. Closeness centrality of node j is defined by:

$$CC_j = \frac{N - 1}{\sum_{k=1; k \neq j}^N d(j, k)}, \quad (5)$$

where N is the number of nodes in the network and $d(j, k)$ is the hop length from node j to node k . Figure 12 shows the relation between closeness centrality and node's degree of topology with different ρ . We use AM-AM networks for this evaluation. Comparing AM-AM networks changing ρ from -0.02 to 0.03 , closeness centrality of high degree nodes increases when the network is connected assortatively. This indicates the increase in the influence of hub nodes on average hop length.

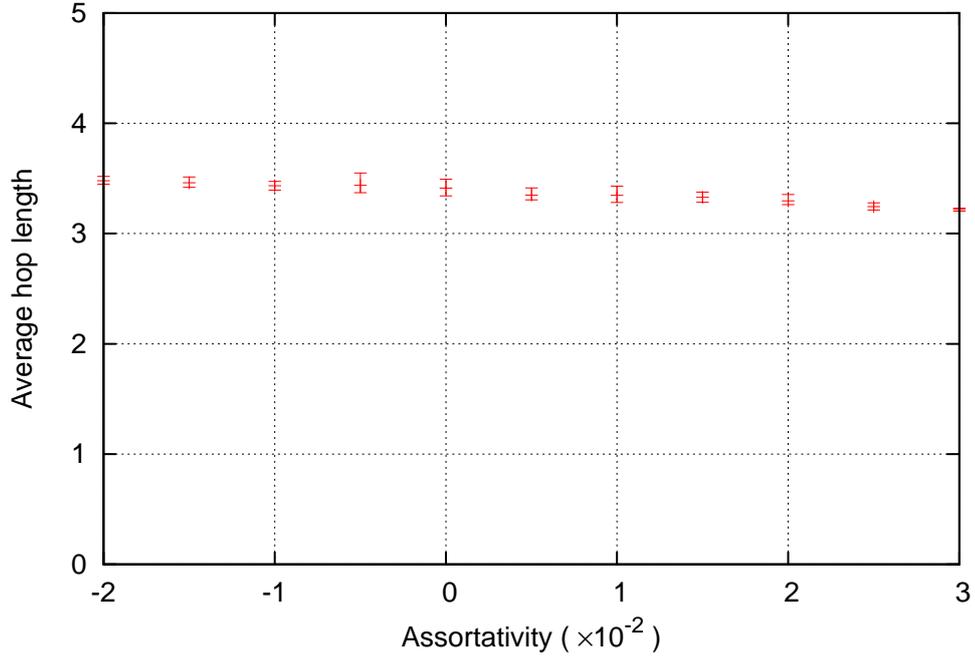


Figure 11: Average hop length in DM-DM networks

3.2.2 Robustness

Figures 13 and 14 show the relation between the number of node failures and the giant component size. A topology with $\rho = -0.02$ is the most vulnerable to selective node failures. This is a natural result from our selective node-failure scenario. Because all edges between networks in case of disassortative connections attach to higher degree nodes, selective node failures soon remove such nodes (namely, edges between networks), which results in fragmentation.

It is worth noting that too high assortativity ($\rho = 0.03$) degrades the robustness of the DM-DM network. This is due to the failures of hub-hub connections. Since a disassortative network is weak for the selective node failures, if almost all of connections between networks are hub-hub connection, the DM-DM network can be disjointed easily.

3.2.3 Information-Diffusion Efficiency

Figures 15 and 16 show that each node's importance on information diffusion. Each node's importance on information diffusion is equally common in different ρ .

Network1	Network2
3	3
14	14
15	15
15	15
19	19
19	19
19	19
25	25

Table 3: Degree of endpoints of all edges between networks when networks are connected assortatively

Network1	Network2
19	3
3	25
19	4
25	3
15	3
3	19
3	19
3	19

Table 4: Degree of endpoints of all edges between networks when networks are connected disassortatively

3.2.4 Edge Betweenness Centrality

Figures 17 and 18 show the edge betweenness centrality of edges between networks. The edge betweenness centrality can be considered as communication loads on edges. Simulations are executed 50 times. Comparing Fig. 17 and Fig. 18, disassortative edges (with $\rho = -0.02$) between networks can distribute communication loads. When networks are connected disassortatively, all edges between networks are connected high-degree nodes with low-degree nodes. Therefore, communication loads are distributed to all edges between networks. On the other hand, topology with $\rho \geq 0$ concentrate communication loads to a few edges. A decrease in edges between high-degree nodes gives further concentration to these edges. Therefore, in the range of $\rho \geq 0$, the topologies other than ρ is the highest show more concentration of communication loads.

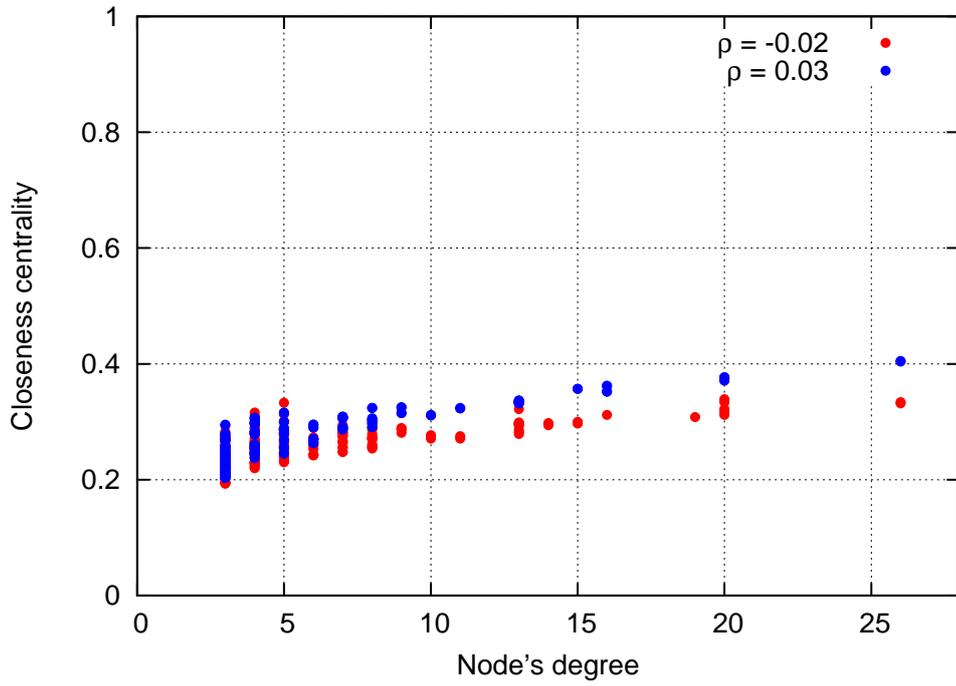


Figure 12: Closeness centrality of each degree (AM-AM network)

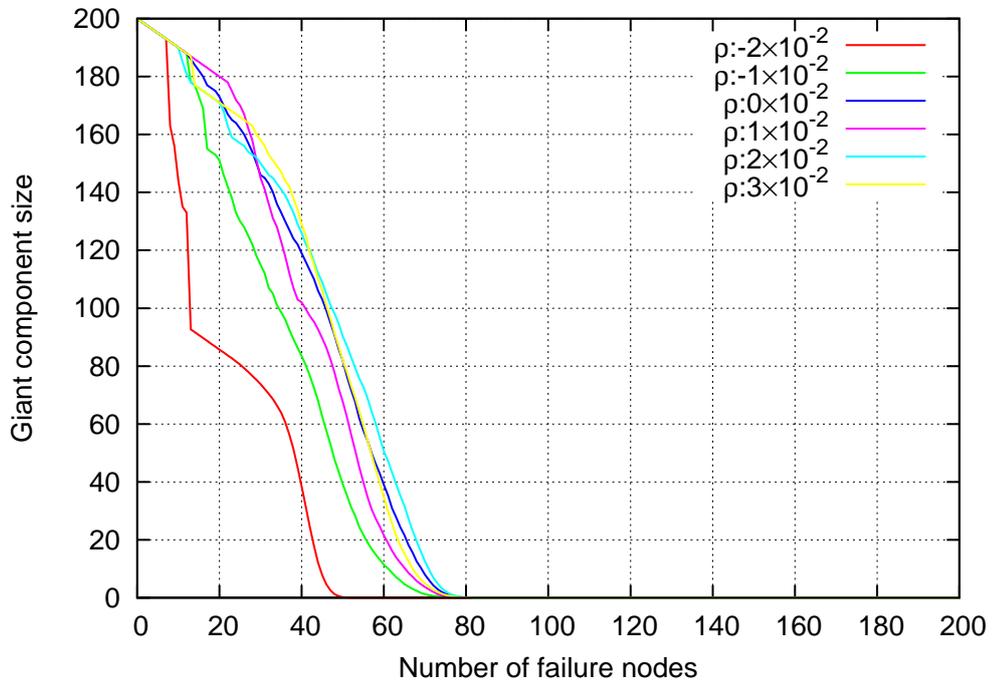


Figure 13: Giant component size in a two-module network with AM topology

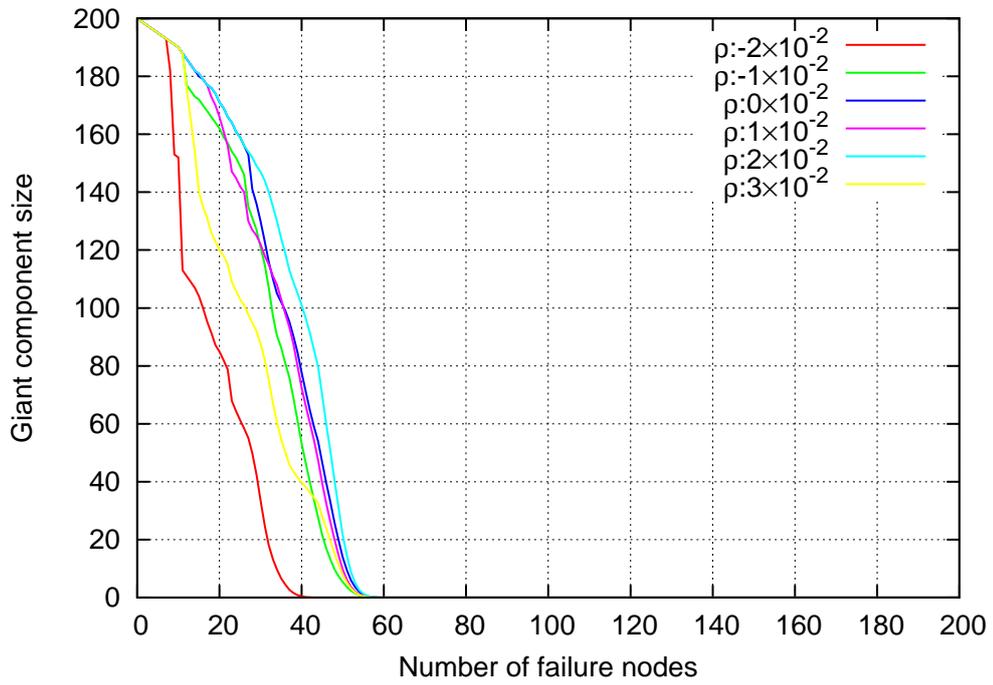


Figure 14: Giant component size in a two-module network with DM topology

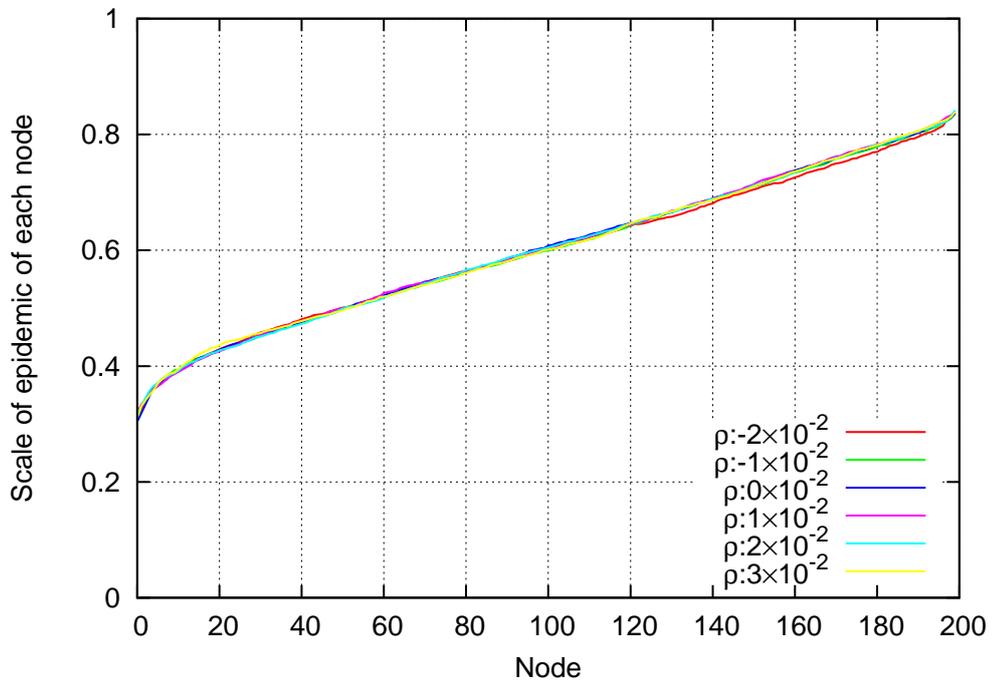


Figure 15: Information-diffusion efficiency in a AM-AM network

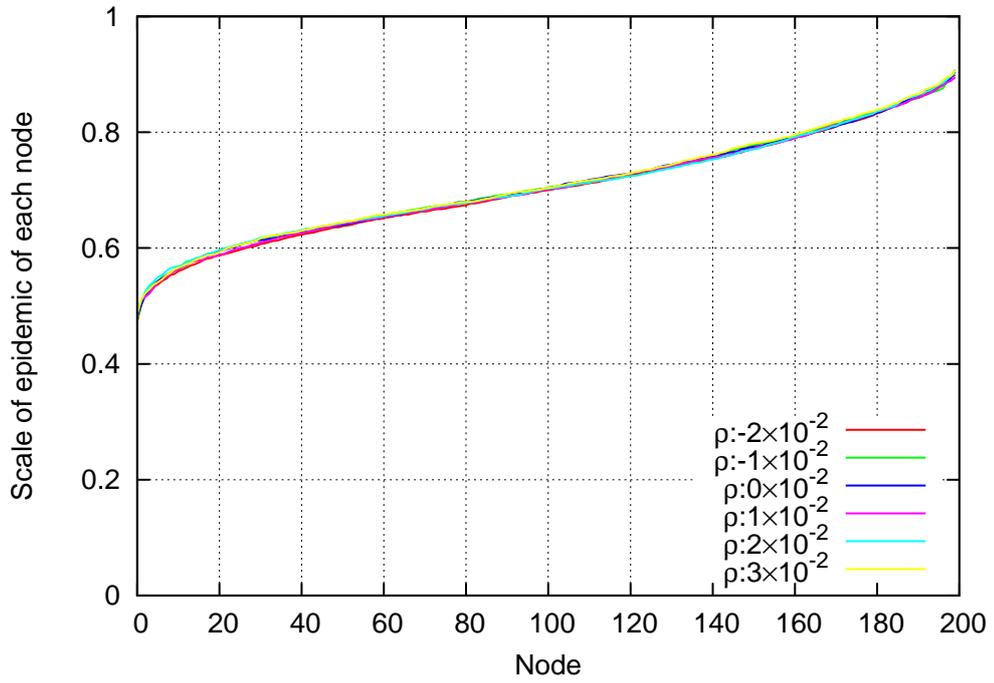


Figure 16: Information-diffusion efficiency in a DM-DM network

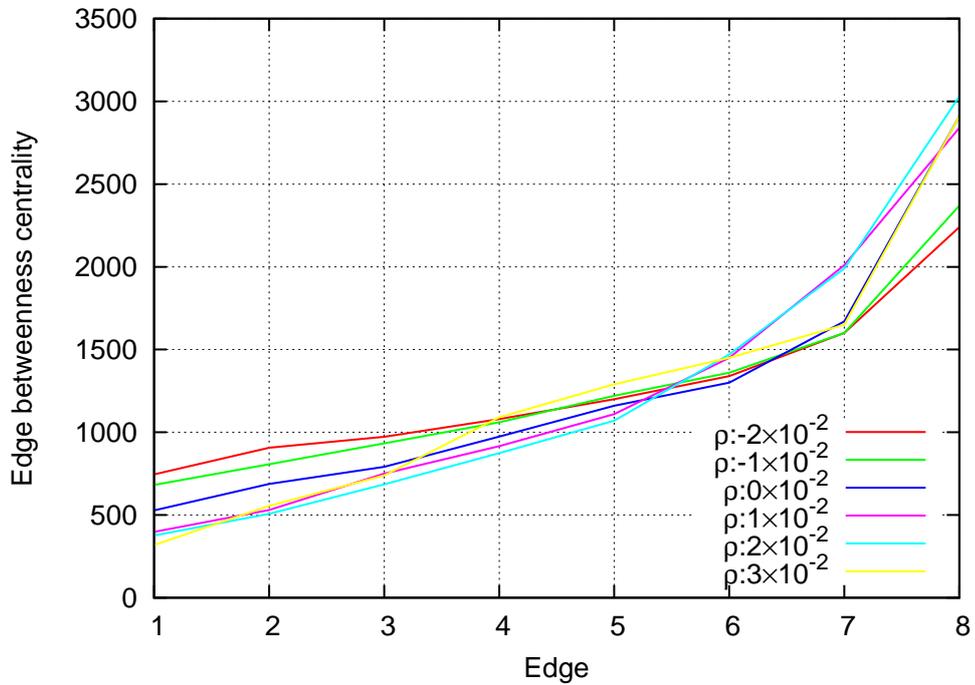


Figure 17: Edge betweenness centrality in a AM-AM network

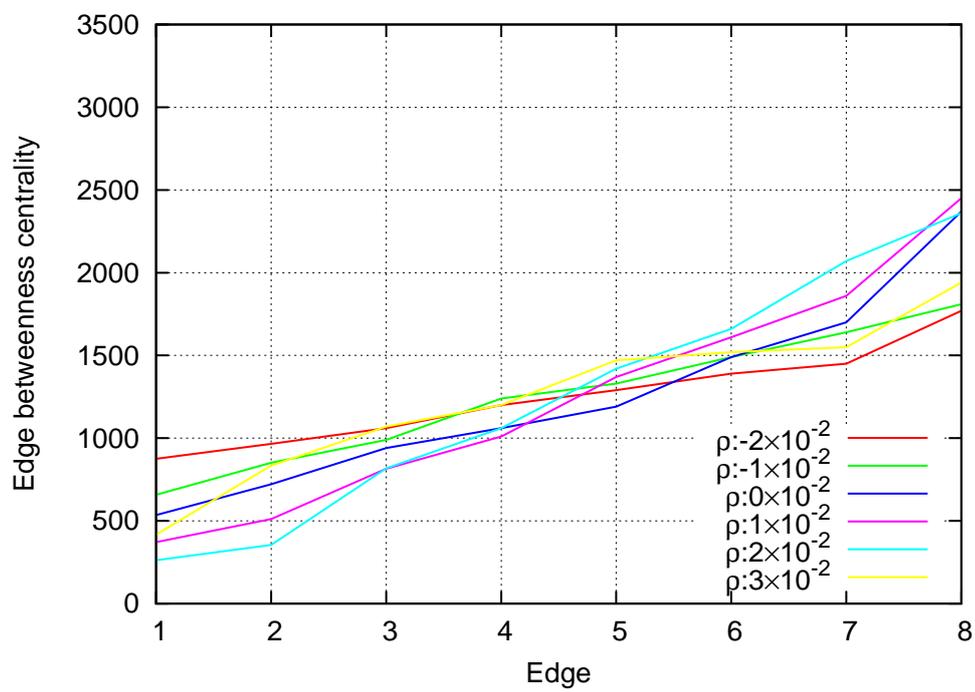


Figure 18: Edge betweenness centrality in a DM-DM network

4 Discussion

4.1 Assortativity's Effect on Robustness and Efficiency

On the assortativity of a single network, we revealed that an increase in the assortativity of a network causes (1) an increase in the hop count, (2) high robustness of connectivity against the failure of high-degree nodes, (3) a low information diffusion efficiency, and (4) a concentration of communication loads on a few edges. An intuitive description for (1), (3), and (4) is that the high assortativity of a network results in the reduction of the shortcut links between the nodes with high degree and low degree as described in Subsections 3.1.1 and 3.1.4. The reason for (2) is sparse connections between nodes that have different degrees. Note that an extremely high assortativity leads to the vulnerable connectivity as mentioned in Subsection 3.1.2. Our results indicate that the assortativity of a network should be within an appropriate range of values.

The range of values in assortativity differs depending on the degree distribution of a network. The network we used in Subsection 3.1 is constructed by the BA model and its assortativity r varies between $-0.67 \leq r \leq 0.58$. We found different ranges of r in networks with different degree distributions. For example, assortativity r has a value within a range of $-0.78 \leq r \leq 0.96$ when we used an Erdős-Rényi random network. When nodes' degrees are uniformly distributed, the range of assortativity r is $-0.78 \leq r \leq 0.98$. Even though the ranges of r differ among these distributions, the change of the assortativity shows a similar effect to the result shown in Subsection 3.1.

On the assortativity between networks, we revealed the following: (1) Assortative connections between networks shortens the average hop length and enhances robustness, (2) disassortative connections between networks distributes communication loads of inter-network links, and (3) disassortative connection between networks is weak against the failures of high-degree nodes. These are seen when both networks that are connected have the assortativity of -0.3 or have the assortativity of $r = 0.3$. Since disassortative networks tend to be disjointed when hub nodes get failed, interconnections of them only with hub nodes reduce robustness.

4.2 Assortativity in Brain Networks

We evaluated the assortativity in inter-connected networks and revealed the effect of it on robustness and efficient. In this subsection, we evaluate the assortativity of human-brain networks and

compare it with our results. We use the data sets of human brain networks from [9]. This data sets contain the weighted connections between the regions of interests (ROI) in a brain. The number of ROIs is 998. We use some thresholds for the weight of connections to define an undirected edge between ROIs.

First, we measured the average assortativity within a ROI and the average assortativity between ROIs with changing the threshold from 0 to 0.055 at an interval of 0.005. We use the Louvain method [23] for identifying the modular structure. Fig. 19 and 20 show the respective results. Assortativity within a module and between modules similarly varies depending on their threshold. When the threshold increases, the assortativity in both results has a trend to decrease. Fig. 21 and 22 show examples of module division when the threshold is 0.005 and 0.045. In each figure, nodes that belong to the same ROI have the same color and symbol.

Edges defined by a high threshold connect important ROIs because the brain enhances edges that are often used [24,25]. As for the assortativity within a module, it shows an assortative mixing pattern when threshold is lower than 0.045, which may mean that not so strong edges are important for robustness against node failures in the human brain while they lead to less efficiency of communication. Though it is just our opinion, this less efficiency in information diffusion prevents unnecessary information diffusion. On the other hand, modules identified with a high threshold show a disassortative mixing pattern. From this result, the important edges in modules are connected so that the average hop length gets shorter. Then, we examine the relationship between the assortativity within a module and its position (e.g., “modules located in the frontal lobes are assortative?”). However we cannot find such a relationship. In Figure 23, assortative modules are presented in red and disassortative modules are presented in blue. As for the assortativity between modules, edges between networks show an assortative mixing pattern with a low threshold. This indicates the human brain can communicate between modules efficiently. On the other hand, modules are connected disassortatively with a high threshold. This may be for the concurrent processing between two modules.

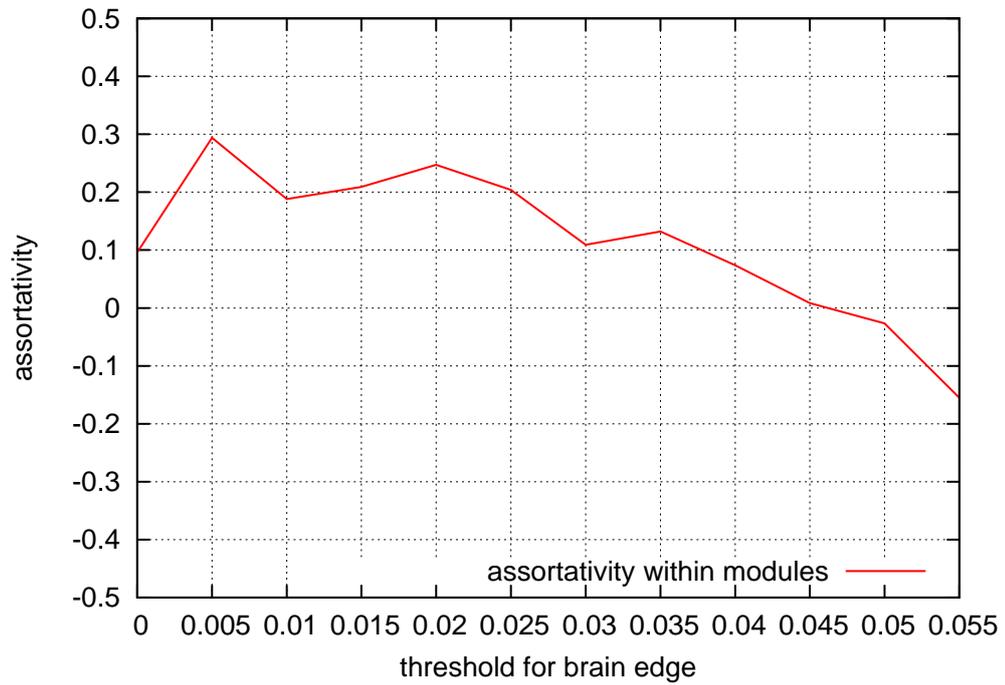


Figure 19: Average assortativity within a module

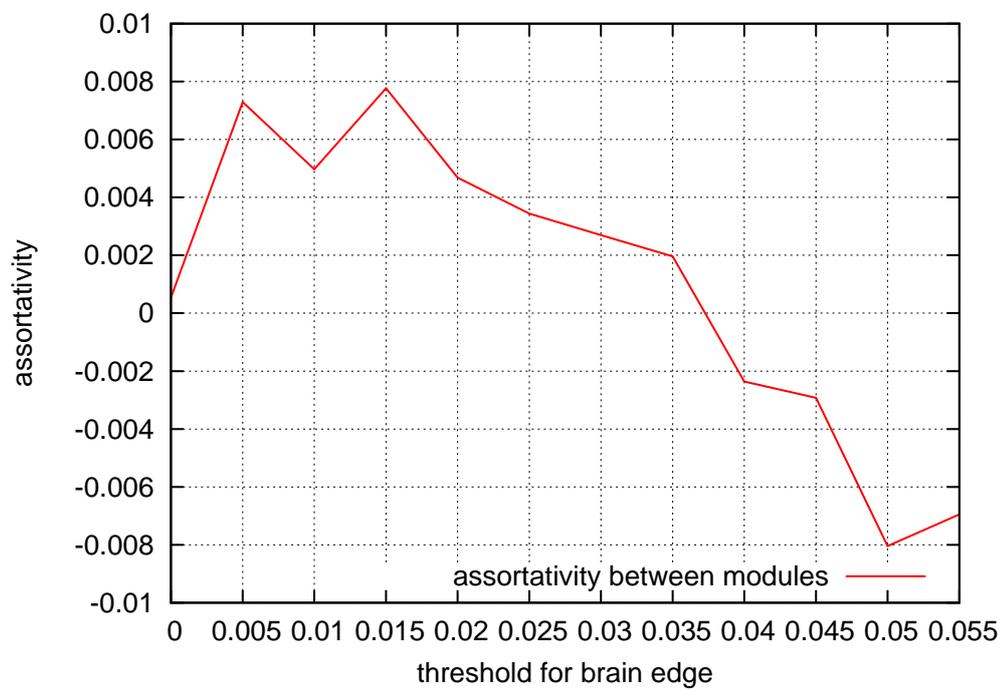


Figure 20: Average assortativity between modules

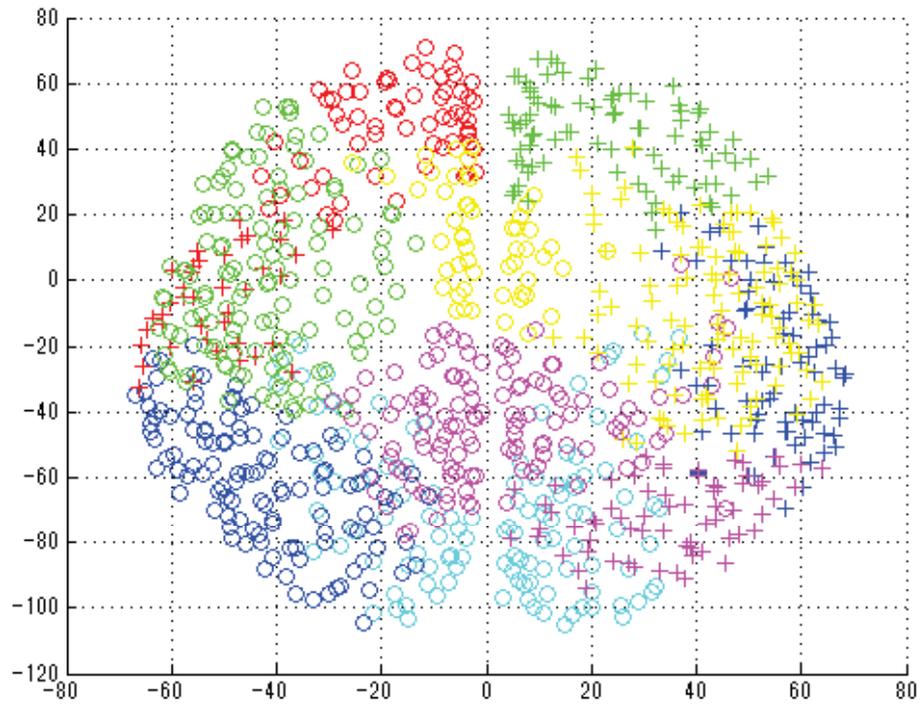


Figure 21: Module division of a brain network with threshold 0.005

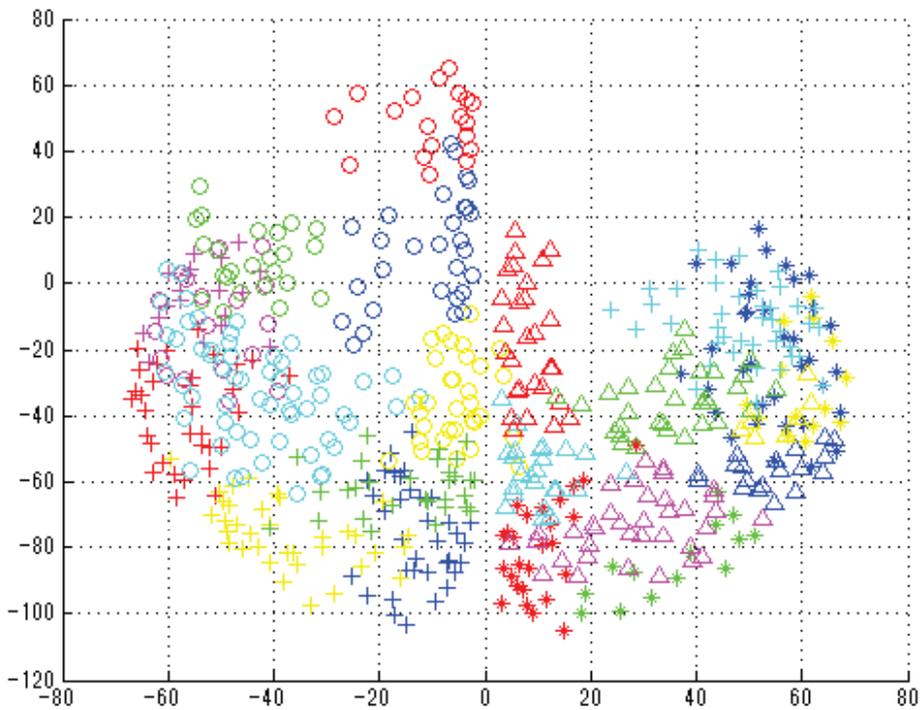


Figure 22: Module division of a brain network with threshold 0.005

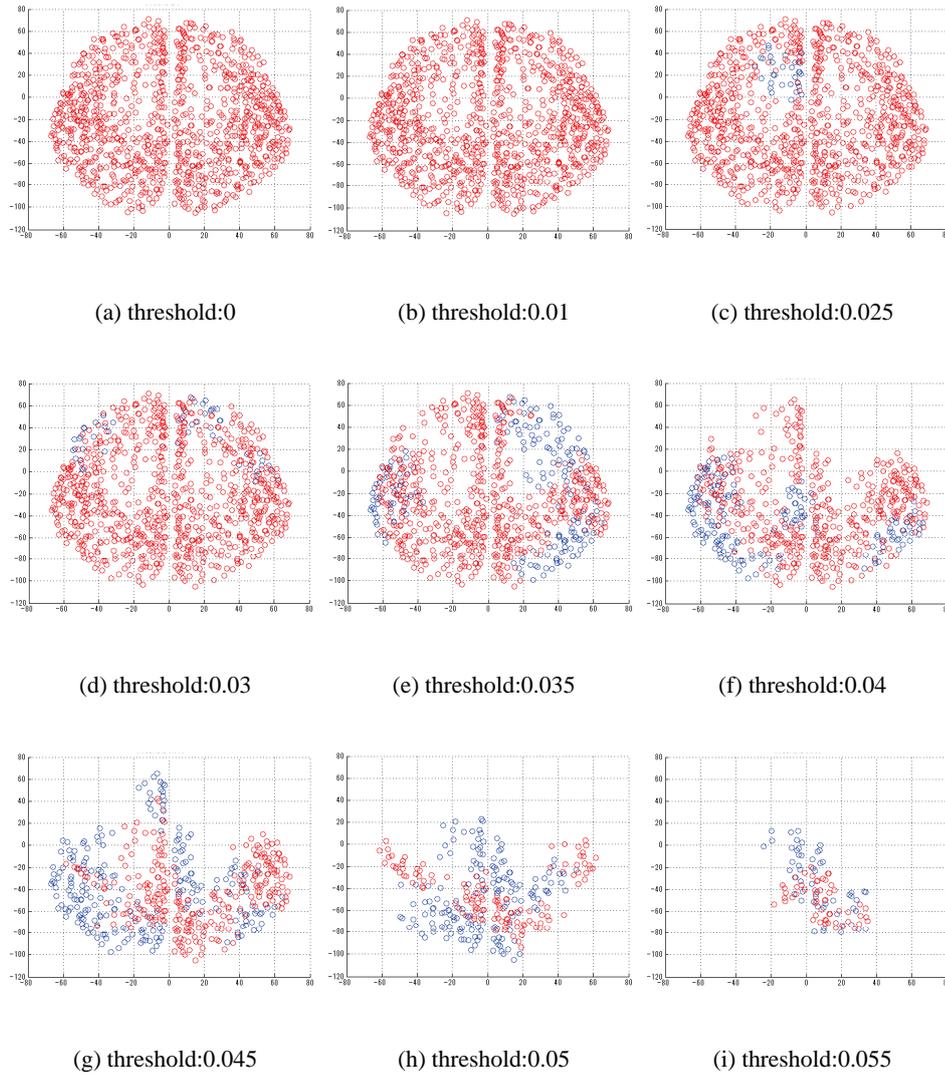


Figure 23: Assortativity distribution in modules in a brain network

4.3 Information-Network Design with Consideration for Assortativity

We showed the effect of the assortativity on the robustness and the efficiency in an inter-connected network. Since we only showed the correlation of the assortativity with the robustness and the efficiency of a network, we cannot answer whether or not a given network is robust and efficient. However, we can answer how to construct a new network so that it has robustness and efficiency. For the network aimed at the information dissemination should be constructed to have a low assortativity. For a robust network construction, high assortativity makes such a network be realized. Furthermore, when the integration of networks is conducted, our results showed that the assortativity between networks can adjust the trade-off between efficiency and load balancing.

For an example of the application of our results, we consider an ad-hoc network which is composed of IoT devices. In this case, the whole network is composed of ad-hoc networks composed of the same devices. Constructing an assortative network has various advantages in this case: high robustness and the inhibition of computer-virus infections. As the defects of an assortative network, a long average hop length and a concentration of communication loads occur. However, as the assortativity of the network is not so high, the average hop length of the network is not so much high. Thus, an appropriate setting of the assortativity is important.

An detailed method for constructing ad-hoc network assortatively is out of the scope of this thesis. However node-deployment techniques or transmission-power control techniques enable it. For example, when more nodes are arranged to be closer to the center of the field, nodes with similar degrees are likely to be connected. Constructing shortcut links between nodes with similar degrees also realizes assortative network, where nodes require the device for directional beams or for long-range omnidirectional transmission.

When networks are connected with each other assortatively, following advantages can be considered: The average hop length shortens by the connection between high-degree nodes. Connections between low-degree nodes bring about the robustness against high-degree nodes. Such failures reflect the depletion of electric power which is caused by the concentration of communication. If networks are connected disassortatively, communication loads are distributed.

5 Conclusion and Future Work

In this thesis, we examined the effect of the assortativity on the robustness and the efficiency in an inter-connected network. On the assortativity of a single network, we revealed that an increase in the assortativity of a network causes (1) an increase in the hop count, (2) high robustness of connectivity against the failure of high-degree nodes, (3) a low information diffusion efficiency, and (4) a concentration of communication loads on a few edges. We show that an increase in assortativity reduces shortcut links in a network. Therefore, too high assortativity causes a harmful effect on the network in regard to the average hop length, robustness and edge betweenness centrality.

On the assortativity between networks, we revealed the following: (1) Assortative connections between networks shortens the average hop length and enhances robustness, (2) disassortative connections between networks distributes communication loads of inter-network links, and (3) disassortative connection between networks is weak against the failures of high-degree nodes. These are the same irrespective of component networks in the inter-connected network.

Although we showed the effect of the assortativity on the robustness and efficiency in an inter-connected network, the results are only for the case that a network is constructed by the BA model. We investigated the assortativity in the case of the Erdős-Rényi random network and the network whose degree distribution is uniformly. However, a inter-connected network that consists of networks that have various degree distributions remains to be investigated. This is our future work.

We discussed how to construct a network so that it has robustness and efficiency. In an actual network, there are various constraints to constructing an assortative or disassortative network. For example, in regard to the ad-hoc network with wireless sensor devices, it is necessary to consider battery and communication distance. Since we did not propose a model for generating a network topology, the proposal of a generation model for assortative or disassortative network and the application in an actual network is also our future work.

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