Master's Thesis

Title

A Study on the Effect of Physical Topology on the Robustness of Fractal Virtual Networks

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Abstract

Virtual networks provided by communication carriers are established for users to achieve specific demands conveniently, such as the traffic demands and quality requirements. Since virtual networks are constructed on the physical topology provided by communication carriers, the performance of virtual networks is limited by the physical topology. Our research group has revealed that the fractality of virtual network, which is basically characterized by high hierarchy and high modularity, is a key to achieving resilience, efficiency, and scalability of information communications.In this thesis, we focus on the effect of physical topology on supporting fractal virtual networks. To support the virtual networks for more convenient information communication, there is a need to improve the capability of the physical topology. We expect that an optimal physical topology can flexibly and robustly support fractal virtual networks with low distance costs of information transferring. Specifically, when the physical topology meets the node failures, the physical topology still can support the virtual networks and perform robustly with holding the connectivity of themselves and can make the virtual networks consume low costs which are the physical distances. We construct a fractal virtual network on several kinds of physical topologies and evaluate the cost-efficiency of physical networks. We find that the scale-free physical topology where the degree distribution follows a power law can keep connected and has less than 1.01 rate of changing on distance cost, while other kinds of physical topologies lose the connectivity or remain high distance costs with more than 1.02 rate of changing on distance cost.

Keywords

Physical Networks

Virtual Networks

Brain Functional Networks

Fractality

Modularity

Hierarchical Modularity

Robustness

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1 Introduction

Nowadays, many people in the world enjoy the Internet with high-speed information transmitting. It is attributed to the actually existent resources of information networks with wise configuring, but the wide applying of virtual networks is crucial to improving the utilization efficiency of physical resources in information networks. Virtual networks are formed by combining hardware and software resources to realize varieties of aimed functions more efficiently. As software is the essential component in virtual networks and strategic to determine the function of a virtual network, it is explicit that virtual networks do not actually exist. It is the virtual network that can expand the functions of physical network resources and improve the efficiency of utilizing physical network resources. Further, with emerging of both SDN (Software Designed Networking) and NFV (Network Function Virtualization), virtual networks will be easily created and heavily used. This trend makes information networks pay more attention to virtual networks. Virtual networks are worth to be developed as the important components both in current information networks and in the future of information networks.

Links in virtual networks represent some relations between nodes or modules as people's attempts but are not the actual routes which information passes by. Thus, the actual routing paths for information transferring between node pairs of virtual networks are provided by physical networks which the virtual networks are embedded on. In other words, the communication in virtual networks is all supported by physical networks, and even the realization of the communication in virtual networks is the realization of the information transfer between corresponding parts in physical networks.

In the real world, virtual networks provided by communication carriers or users are established for users to achieve variously specific demands conveniently, such as the traffic demands and quality requirements. Once users are allowed to access networks provided by ISPs or communication carriers, they can acquire the services provided by resources of physical topologies supplied by ISPs or communication carriers. Though ISPs or communication carriers provide a large scale of information network resources, the parts working as the same function, which users pay for or which users can use for their specific communication demands, always have much smaller scales than the whole physical topologies resources. It can be said that virtual networks can select any parts of physical networks to become an useful and effective networks utilized by people so that virtual networks are flexible.

Recently, fractality emerges from the organizations of networks on different fields as biology, technology, and sociology. Fractality is a principle that describes organizations which share self-similarity on all length scales [2]. It is also discovered in brain functional networks. With more observations on biological networks, it indicates that there are few hub-hub connections, also being called "disassortativity", in fractal networks. Hubs are the most connected nodes in networks. In Ref. [2], it shows that the fractal property of networks significantly increases the robustness against targeted attacks because the hubs are more dispersed in the network. It means that fractal topologies can hold the connectivity, where each node in topology has, at least, one or more route to reach all other nodes, after removing more hubs than non-fractal topologies. Moreover, fractality leads to rich modules and hierarchical structures due to its feature of self-similarity on all length scales. In brain networks [1] which are fractal with rich modules and hierarchical structures, the modularity leads to the segregation functions that each module can perform a specific function inside itself, and the hierarchy leads to the integration functions that different modules perform a specific function together between modules. It is even said that fractality existing in brain networks contributes to the excellent performance on achieving brain functions.

Due to lots of advantages of fractality, it can be expected to utilize such a topological property of fractality into virtual networks for information networks to perform as like in the case of biological networks. Ref. [3] shows that the fractal topologies can keep reachability between modules against the various types of node failures and relax the traffic concentration. This result provides the potentiality of raising information networks performances by applying fractal virtual networks. Therefore, virtual networks with fractality is an expected subject and is also a developing trend which is potential to promote the advanced performance for information networks.

However, we do not suggest to bring fractality into physical network topology of information networks. The reason is that fractality leads to what physical topology cannot support virtual networks efficiently enough. It is because fractality has an effect on the long diameter which is the longest distance of the shortest path among all node-pair distances in the whole topology. As known, the types of virtual networks on the structure are diverse and with the development of society, more and more virtual networks will be added to information networks. To support varieties of virtual networks well at the same time, the most important target of physical network is to realize the rapid information transferring, which we call "efficiency". Inversely, the self-similarity on all length scales of fractality means that the diameter of fractal topology is exponential times as the diameter of the smallest module. As the scale of information networks grows day by day to meet human's demands, the diameter of fractal topology will exponentially increase with the growing scale. So fractality violates the requirement of high efficiency so that it is not suggested to bring fractality into physical topology for information networks.

As introduced above, virtual networks are constructed on physical networks. So no matter how functional the virtual networks are, they need physical networks under them to support their perfect performances. That is, the physical topology for information networks plays an essential role in information transfer. As known, besides physical materials for signals transferring and routing algorithms, the structure and the way of connecting devices and cabals, which are called nodes and links respectively, basically affect the performances of networks. These characteristics on structures are called topological properties, and they can be used to describe all kinds of networks on patterns of structures. Among many classifications of networks, according to whether owning the real spaces, they can be divided into two types of networks, virtual networks, and physical networks. In information networks, physical network refers to the network are comprised by the infrastructure including cables, routers, servers, etc. On the other side, virtual networks in information networks are constructed according to varieties of people's demands based on physical networks. They can use all of the physical resources of physical networks, and also, can partially use the physical resources of physical networks according to their expected functions. They usually use nodes of physical networks as nodes and connect nodes by links which are determined by the actual demands of communications. Hence, it is necessary to discover or even design a physical topology for physical networks supporting virtual networks with diverse changing, such as the input of fractality.

On biological complex networks, brain networks are similar to information networks that there are structural connectivity and functional connectivity where structural connectivity is obtained from anatomical nervous networks and functional connectivity is mapped according to the correlation of brain regions. In Ref. [4], it indicates that brain network organization including both structural connectivity and functional connectivity is economical that there is a trade-off between cost and efficiency, where cost refers to the link density and efficiency refers to all the length of the shortest paths between each node pair. Further, functional connectivity is always changing to achieve specific functions of the brain while keeping economical under the supporting of structure connectivity which changes less than functional connectivity. Information networks are somehow similar to the mechanism of brain networks that virtual networks are supported by the physical network whose topology changes less than virtual networks.

Inspired by brain networks, we consider that we can have a physical topology performing as perfect as brain structure connectivity. Therefore, to bring both the advanced fractal property and learning from advanced brain networks, we focus on investigating the optimal type of physical topology to support fractal virtual networks flexibly with low distance costs. And to hold the robustness of fractal virtual networks, we also stipulate this type of physical topology that it can support fractal virtual networks robustly with low distance costs and less changing on distance costs when meeting with node failure.

This thesis is organized as follows. In the Section 2, we define designing criteria of physical topology to support fractal virtual networks; Section 3 induces an optimal problem for physical topology supporting fractal virtual networks with designing criteria; Section 4 provides an optimization algorithm for leading to the result of this study; Section 5 obtain a result of this study by numerical evaluation following optimizing formula and applying the optimization algorithm. Finally, we conclude the study work in Section 6 .

2 Connectivity Requirements of Physical Topology to Support Fractal Virtual Networks

Neural connectome of human brains can be modeled and expressed by brain networks or brain graphs which are composed of nodes and edges on the basis of the activities of brain neural systems. A node of brain networks is a portion of the system that is separable from the other portion of the system in some way [4]. There are many ways to defining nodes in brain networks. For instance, a ROI (Region Of Interest) of the cortex can be defined as a node in brain networks. On the other hand, an edge of brain networks represents a relation between two nodes connected by it and the defining ways are more open to a variety of legitimate choices. In structural brain networks, an edge is derived from anatomical connectivity which represents actual existing of axonal connection between gray matter regions anatomically. In functional brain networks, an edge is defined by measuring two processes behaving similarly over time. The correlative coefficient is a measure of functional connectivity for estimating the similarity of behavior between two areas. With a set thresholding of correlative coefficient, functional networks can be constructed, whose edges have correlative coefficient larger than the thresholding. Therefore, it is obvious that edges in structural brain networks actually occupy physical spaces and exist anatomically, and edges in functional brain networks do not occupy physical spaces and represent similar behaviors over time dynamically.

In brain networks, activities of functional networks are supported by the connectivity of structural networks. Experiments have shown that the structural and functional connectivity of brain networks have high positive correlations with each other [5]. In other words, if two brain areas are strongly connected anatomically, it is likely that they will share high functional correlations too. Moreover, brain networks scientists are paying much attention on inferring both structural connectivity from functional connectivity [6] and functional connectivity from structural connectivity [5] in several disciplines. All these works indicate that structural brain networks provide their anatomical resources for realizing functional conscious tasks, strongly based on the demands of their functional brain networks. Ref. [1] reveals that brain networks negotiate the trade-offs between wiring cost and topological efficiency value (as seen in Fig. 1). In graph theory, wiring cost refers to the link density of brain networks and topological efficiency refers to the total distances which all node pairs take to realize the signals transmitting between them. To achieve a

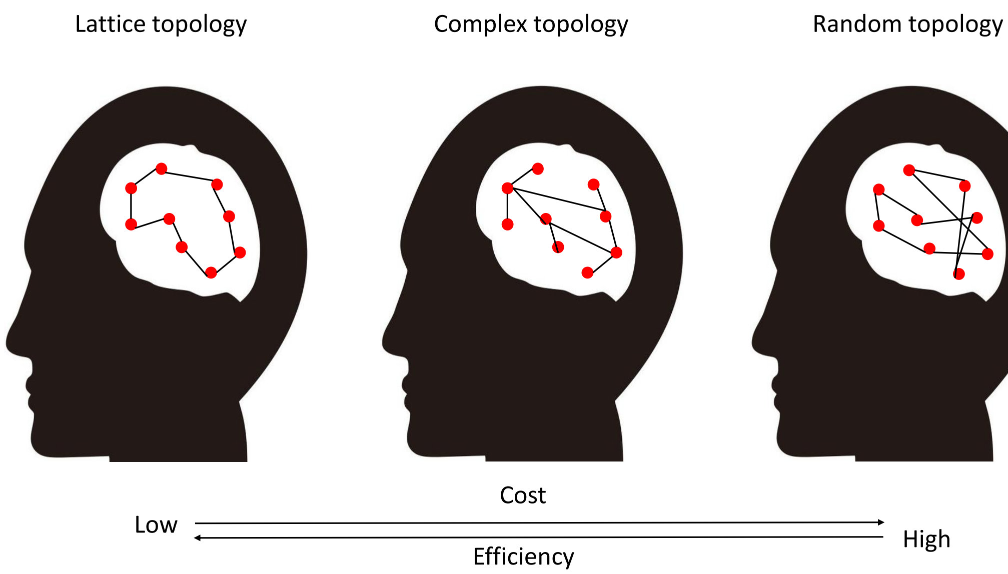

Figure 1: Complex brain network with structures is economical to negotiate on the trade-off between cost and efficiency [1].

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high efficiency maximized by a random topology, brain networks will contain to more edges between long-distance nodes to minimize the total distances. On the other side, to achieve a low wiring cost minimized by lattice-like topology, brain networks will decrease the number of edges to minimize the energy of building edges. Brain networks always negotiate their topologies for functional conscious tasks between these two extreme cases.

In information networks, there are physical topology and virtual network as same as the relation between structural and functional networks in the human brain. Physical topology occupies physical spaces and virtual networks do not occupy physical spaces. Nodes in virtual networks use the resources of the nodes of physical topology, and links in virtual networks do not exist spatially and represent the communication demands or relations, etc. Due to the advanced performances of brain networks, it is beneficial for information networks to learn from brain networks. According to the characteristics of structural networks supporting functional networks in the brain, we extract some criteria of physical topology as follows for supporting fractal virtual networks and it is feasible to design a physical topology following such criteria because information networks and brain networks can be all measured by graph theory.

2.1 Flexibility

As brain functional networks, which embedded on the brain structural network, can change their topologies into many kinds of networks for the conscious tasks, it can be said that brain structural networks can flexibly support functional networks with topological changing on the basis of functional tasks. The capability of flexibly supporting functional networks of structural networks in the brain is reflected on following the economy of brain network organization, the trade-offs between a wiring cost and efficiency, no matter what types of functional networks change to.

Inspired by this view, we redefine the flexibility of physical topology when supporting fractal virtual networks in the field of information networks. A physical topology with flexibility should support many fractal virtual networks with a low average distance cost. The capability of flexibly supporting fractal virtual networks of physical topology is reflected on keeping low distance cost which is the basic requirement of information networks, no matter what structures of fractal virtual networks have.

2.2 Robustness

As brain functional networks of normal human, which embedded on the brain structural network, can precisely realize varieties of conscious functions, it can be said that brain structural networks can robustly support functional networks working without faults. The capability of robustly supporting functional networks of structural networks in the brain is reflected on successfully realizing the conscious functions, no matter what conditions brain network system meets. Even in disease states, such as Alzheimer's disease [7, 8], brain networks still keep connected and potentially provide important biomarkers for psychiatric and neurological disorders [9].

Inspired by this view, we redefine the robustness of physical topology when supporting fractal virtual networks combining to the characteristics of fractal networks in the field of information networks. Among many characteristics of fractal networks, robustness against hub failures is significantly predominant in network science. Considering this characteristic, we define that a physical topology with robustness should support fractal virtual networks with hub failures by holding the connectivity and with a low distance cost. Therefore, the capability of flexibly supporting fractal virtual networks of physical topology is reflected on holding the connectivity which is the basic requirement for supporting fractal networks, but also keeping a low distance cost which is the basic requirement of information networks, when fractal virtual networks meet with hub failures.

2.3 Topological Properties

Brain networks show a complex topology for themselves to be able to negotiate the topology following the trade-offs economy. Based on it, the type of a physical topology which can support fractal virtual networks flexibly and robustly should be one of the types of complex networks.

For this view, we investigate several types of networks and conclude them with main feature illustration as follows.

- Small-world networks: Networks which have large clustering coefficient and short path between each pair of two nodes and follow equations (1)(2)(3) [10]. In these equations, small-world network has a clustering coefficient, C, and greater than the clustering coefficient of a random network, C_τ, and the path length, L, is about the same as the path length of a random network, L_τ, therefore a small-world scalar $\sigma > 1$.

- Scale-free networks: Networks whose degree distributions follow power-law of the form $P(k)\ k^{-\gamma}$, where k is a degree value [11].

- Random graphs: Networks which are obtained by starting with a set of n isolated nodes and adding links between them randomly. And they have arbitrary degree distribution due to the random connecting.

- Lattice graphs: Networks which are regular on the structure and embedded in some Euclidean space R^n.

- Fractal networks: Networks whose structures look the same on all length scales on geometric theory [2].

$$\sigma = \frac{\frac{C}{C_\tau}}{\frac{L}{L_\tau}} > 1 \tag{1}$$

$$\gamma = \frac{C}{C_\tau} > 1 \tag{2}$$

$$\lambda = \frac{L}{L_\tau} \sim 1 \tag{3}$$

Therefore, an optimal physical topology will be one type of above networks or mixed type of them.

3 An Optimization Problem of Physical Topologies with Capacity and Connectivity Requirements

3.1 Literal Expression of Optimization Problem

Designing Criteria of Physical Topology for Supporting Fractal Virtual Networks are established on three aspects which include two features of supporting performances, flexibility and robustness, and one limited topological type. The multiple criteria lead to that this study is an optimal problem. There are two reasons which lead to this study becoming an optimal problem. The one is that the two features of supporting performances exist at the same time so that it is difficult to find or design a physical topology which can perform best at both two aspects. The other one is that the number of types of physical topology candidates cannot be fixed so that we can explored continually designed methods of physical topology are improved gradually. Therefore, it is an approach to induce and this study into an optimal problem.

First, we start with designing criteria introduced in the section 2 to define the optimal problem. As the topological properties will be a result of the type of the found or designed physical topology, the criterion of “Optimal Physical Topology Should Be with Topological Properties in the Field of Complex Networks” is set as a designing premise of physical topology. And the other two criteria on flexibility and robustness are set as the optimized target objects.

Second, after having the optimized target objects, we consider the measures of optimized target objects, flexibility, and robustness. Based on the definitions of capabilities both on flexibility and robustness, we draw up the measure for each object. For flexibility, average distance cost. which is the sum of the distance lengths of the shortest paths between all node pairs, measures that the lower it is, the more flexible the physical topology is to support fractal virtual networks. For robustness, changing rate of the distance cost, which compares the cost of physical topology with connectivity under failures to origin cost before failures, measures that the smaller the rate is, the more robust the physical topology is to support fractal virtual networks with hub failures.

Thus, the optimization problem for this study is to minimize the average distance cost and the changing rate of the distance cost with both the constraint of holding the connectivity under hub failures and the constraint of the definition of distance cost which is the sum of the distance lengths of the shortest paths between all node pairs. We provide a mathematical expression for this optimal problem and illustrate it in the next subsection.

3.2 Mathematical Expression of Optimization Problem: Problem Formulation

3.2.1 Settings Prepared for Formula Expression

On notations, I introduce the definitions at 4 aspects.

First, formulate the notations on graph definitions as follows.

$G = (V, E, W)$**:** The unified graph definition for physical topology. In different cases, it will bring on subscript or superscript, such as $G_{topology}$.

- V: The total nodes set. We use $v(v = 1, 2, ..., |V|)$ to label a node.
- E: The total links set. We use $e(e = 1, 2..., |E|)$ to label a link.
- W: The lengths set of total links for this study. We use $w_e(w_e = w_1, w_2, ..., w_{|E|})$ to label a length of a link.

$G = (V, E)$**:** The unified graph definition for virtual network. In different cases, it will bring on subscript or superscript, such as $G_{virtual}$.

$M_{topology}$**:** The total physical topologies set.

$M_{virtual}$**:** The total virtual networks set.

R_{node}**:** The failed nodes set.

Second, formulate the notations on traffic demand as follows. All demands arise from the virtual networks.

D**:** The total number of traffic demands. On the present phase, we simply use it to express the end node pair of traffic demand.

d**:** A label to express a traffic demand, $d(d = 1, 2, ..., D)$

Third, formulate the notations on the paths as follows. All paths is in the physical topology to achieve the demands of virtual networks.

P_d**:** The total number of candidate paths for demand d.

P_{dp}**:** A path for demand d.

p**:** A label to express assigned path, $p(p = 1, 2, ..., P_d)$.

$\mathbb{P}_d$**:** The total paths set for demand d, $\mathbb{P}_d = (P_{d1}, P_{d2}, ..., P_{dP_d})$

At last, formulate the notations on others

f**:** It is used as a subscript to express that the virtual network is fractal.

$*$**:** It is used as a superscript to express that the physical topology is under the node failure or the virtual network is supported by a physical topology under the node failure.

I introduce two parameters which are given parameters and a variable.

We have given parameters as follows.

- R_{node}
- $M_{virtual}$ and $M_{topology}$
- $G_f = (V_f, E_f) \in M_{virtual}$ is the fractal virtual network.

And there is only one variable in this formulation as follows.

- $G_x = (V_x, E_x, W_x) \in M_{topology}$

For the target formula, I divide it into two items which are about flexibility and about robustness.

For flexibility,

$$\delta_{e_x dp_x} = \begin{cases} 1 & if\ link\ e_x\ belongs\ to\ path\ p_x\ for\ demand\ d \\ 0 & otherwise. \end{cases}$$

$$\begin{aligned} DistanceCost(d) &= \sum_e \delta_{e_x dp_x} \times w_{e_x} && (4) \\ DistanceCost(G_{virtual}) &= \sum_d DistanceCost(d) && (5) \\ DistanceCost(M_{virtual}) &= \sum_{G_{virtual} \in M_{virtual}} DistanceCost(G_{virtual}). && (6) \end{aligned}$$

For robustness,

$$\begin{aligned} DistanceCost(d_{f^*}) &= \sum_{e_x^*} \delta_{e_x^* d_{f^*} p_x^*} \times w^*_{e_x^*} &(7)\\ DistanceCost(G_{f^*}) &= \sum_{d_{f^*}} DistanceCost(d_{f^*}) &(8)\end{aligned}$$

$$\begin{aligned} Co-DistanceCost(d_{f^*}(G_f)) &= \sum_{e_x} \delta_{e_x d_{f^*} p_x} \times w_{e_x} &(9)\\ Co-DistanceCost(G_f, D_{f^*}) &= \sum_{d_{f^*}} Co-DistanceCost(d_{f^*}(G_f)). \end{aligned}$$

3.2.2 Result Formula with Constraints

The result of problem formulation is as follows.

$$Minimize \quad \frac{DistanceCost(M_{virtual})}{|M_{virtual}|} + \frac{DistanceCost(G_{f^*})}{Co-DistanceCost(G_f, D_{f^*})} \tag{10}$$

Constraint 1: The shortest path

$$\begin{aligned} DistanceCost(d) \le \sum_{e_x} \delta_{e_x d i_x} \times w_{e_x},\\ for\ each\ path\ i_x,\ 1 \le i_x \le P_d &\quad (11)\\ DistanceCost(d_{f^*}) \le \sum_{e_x^*} \delta_{e_x^* d_{f^*} i_x^*} \times w^*_{e_x^*},\\ for\ each\ path\ i_x^*,\ 1 \le i_x^* \le P_{df^*} &\quad (12)\\ Co-DistanceCost(d_{f^*}(G_f)) \le \sum_{e_x} \delta_{e_x d_{f^*} i_x} \times w_{e_x},\\ for\ each\ path\ i_x,\ 1 \le i_x \le P_d &\quad (13)\end{aligned}$$

Constraint 2: Holding the connectivity under the failure

$$\begin{aligned} D_f &= \frac{|V_f|(|V_f|-1)}{2} &(14)\\ D_{f^*} &= \frac{|V_{f^*}|(|V_{f^*}|-1)}{2} &(15)\end{aligned}$$

$$\mathbb{P}_{d_f} \neq \phi,\ \ for\ \forall d_f,\ 1 \le d_f \le D_f \tag{16}$$

$$\mathbb{P}_{d_{f^*}} \neq \phi,\ \ for\ \forall d_{f^*},\ 1 \le d_{f^*} \le D_{f^*}. \tag{17}$$

4 Mathematical Formulation to Decide Physical Topology

Following the optimization problem formula, if we solve the optimization problem by inputting a physical topology which leads to the minimum value of the formula (10) and meeting the constraints (12) – (17), we obtain a minimum value of the formula (10) as a result. However, the result of this study should be a type of physical topology and not a value. Therefore, we propose an optimization Algorithm on solving the optimization problem to obtain a type of physical topology which will be the result of this study.

First, we try to find the solution from the parameters. It can be seen that the given parameters are the conditions of this study which are fixed, but the variable which represents physical topology candidates are not fixed. It indicates that only variable which is a physical topology and the result of formula (10) under the constraints (12) – (17) can be changed and others are all fixed once given a definition in the optimal process.

Second, we find that the result of formula (10) under the constrains (12) – (17) is determined by the variable of physical topology with all other fixed parameters. It indicates that a physical topology as a variable input has only one result of formula (10) under the constrains (12) – (17) and this result cannot be minimized under the same physical topology as a variable.

According to above two findings, we propose an optimization Algorithm to achieve the study goal following the problem formula as follows.

Step 1 Find or generate the different types of physical topology candidates as many as possible.

Step 2 Input each physical topology candidate as a variable into the formula to obtain a value of formula under its constraints for each physical topology candidate.

Step 3 Compare these values of formula to find a minimum value and its variable physical topology candidate.

Step 4 Conclude that the type of physical topology which leads to a minimum value of formula under the constraints is the optimal among these physical topology candidates.

5 Numerical Evaluations on the Capacity and Connectivity of Physical Topologies

We do the numerical evaluation following the formula (10) and its constraints from (12) to (17), which can express the contents of this study on a mathematical method. Here, we set situations to meet the conditions for the numerical evaluation and show the results of the numerical evaluation.

5.1 Simulation Environment

5.1.1 Models of Both Physical Topologies and Virtual Networks for Numerical Evaluation

Models are as follows and will be the objects of the numerical evaluation. The supporting combination relations can be seen as Table. 5. We also provide the generating methods for all these models.

- Full mesh networks
- Fractal networks
- WS (Watts-Strogatz) model which is small-world and represents a small-world network
- BA model which is scale free and represents a scale-free network
- Random graph
- Lattice graphs
  - Ring: 1D lattice graph
  - Grid: 2D lattice graph
  - Cube: 3D lattice graph

Method on Generating a Full Mesh Model

The method of generating a full mesh network is to make each node connected to all other nodes.

Method on generating a fractal model [2]

- Introduction of fractal model proposed in [2]

In general, fractal topology indicates that hubs which are the most connected nodes tend to connect to nodes with small degree. In contrary, non-fractal topology indicates that hubs tend to be connected with other hubs. In this reference, topologies can be modularized by directed diameters using box algorithm. The module after modularization is regarded as a box, and the most connected node in the box is the hub of the box. Fractal topologies are suggested to be generated by time due to its scale-free feature.

Authors propose a mathematical model as equations(18)(19)(20) for generating fractal topologies,

$$\tilde{N}(t) = n\tilde{N}(t-1), \ n > 1 \tag{18}$$

$$\tilde{k}(t) = s\tilde{k}(t-1), \ a > 1 \tag{19}$$

$$\tilde{L}(t) + L_0 = a(\tilde{L}(t-1) + L_0), \ a > 1 \tag{20}$$

Where$t$ represents the generation time, and $\tilde{N}$ represents the total number of nodes, and $\tilde{k}$ represents the degree of each node, and $\tilde{L}$ represents the diameter which is the longest distance among all the shortest paths between all node pairs, and n,s,a are all constants.

Then, a fractal topology can be generated following the mathematical model, and the sketchy procedure is as follows.

Step 1 Set an initial structure and define this state as time $t = 0$.

Step 2 Set a boxsize l which is the diameter of each box.

Step 3 Regard each node as a box, and add nodes with links into box so that the boxsize of each boxes reachs l.

Step 4 Rewire the connections of hubs between boxes to ensure that there is no hub-hub connection.

Step 5 Set the current state as time $t = t + 1$.

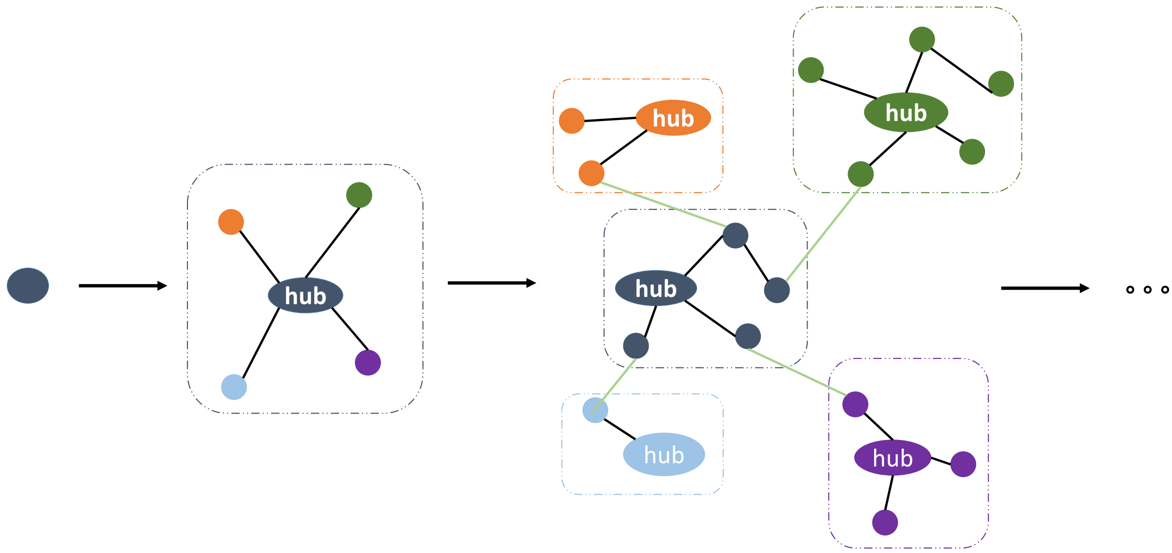


Figure 2: MODE2 (fractality: hub-hub repulsion). The time generating pattern of fractal topologies.

Step 6 Return to Step 3 until the scale of topology has reached what expected to.

The topology generated by above procedure is a fractal one, called "MODE2" in Ref. [2]. We can see a simple example of the pattern of the generation with a time of a fractal topology as Fig. 2.

With a little modifying in the procedure as follows, we can also obtain a non-fractal topology by this mathematical method.

Step 1 Set an initial structure and define this state as time $t = 0$.

Step 2 Set a boxsize l which is the diameter of each box.

Step 3 Regard each node as a box, and add nodes with links into box so that the boxsize of each boxes reachs l.

Step 4 Check the connections of hubs between boxes to ensure that there are all hub-hub connections.

Step 5 Set the current state as time $t = t + 1$.

Step 6 Return to Step 3 until the scale of topology has reached what expected to.

The topology generated by above procedure is a non-fractal one, called "MODE1" in Ref. [2]. We can see a simple example of the pattern of the generation with a time of a non-fractal topology as Fig. 3.

By the two above generating procedures, both fractal and non-fractal generated topologies follows the mathematical equations(18)(19)(20).

- The Minimal Model

There is the simplest model, called "the minimal model", as a type of generating. The generating procedure is as follows with more specific parameters.

Step 1 Set a 5-nodes star structure as an initial structure and define this state as time $t = 0$.

Step 2 Set a box size $l = 3$ which is the diameter of each box.

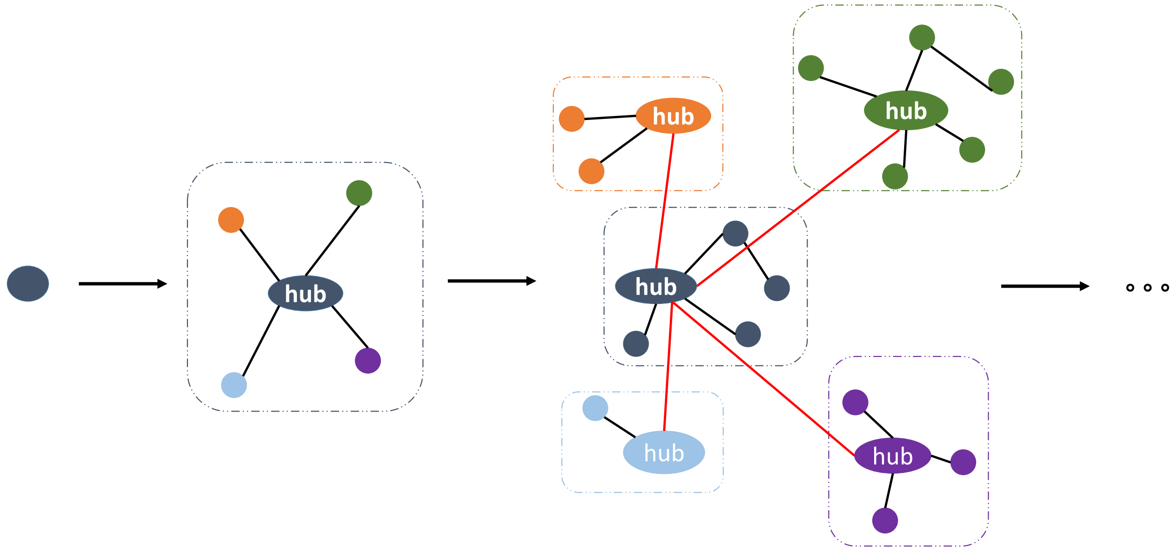


Figure 3: MODE1 (non-fractality: strong hub-hub attraction). The time generating pattern of non-fractal topologies.

Step 3 Set a parameter $m = 2$ which means that add new nodes to the existing node with the number as the twice number of the degree of it.

Step 4 Regard each existing node as a box and add new nodes to it.

Step 5 Rewire the connections of hubs by that choose a less connected node from each box instead of the hubs for connecting between boxes.

Step 6 Set the current state as time $t = t + 1$.

Step 7 Return to Step 3 until the scale of topology has reached what expected to.

Step 5 is significant to determine the fractal feature of generated model. If we rewire the connections between boxes with probability, e, the generated model can be controlled on the ingredient of fractality.(Seen as Fig.4)

Moreover, we ensure that all fractal topologies used in the evaluation had been verified on the fractality by the fractal dimension with applying a box-covering algorithm [12–14].

Method on generating a WS (Watts-Strogatz) model [10]

Watts and Strogatz proposed a one-parameter model that interpolates between an ordered finite-dimensional lattice and a random graph. The algorithm [11] behind the model is as follows.

(1) Start with order: Start with a ring lattice with N nodes in which every node is connected to its first K neighbors ($K/2$ on either side). In order to have a sparse but connected network at all times, consider $N \gg K \gg ln(N) \gg 1$.

(2) Randomize: Randomly rewire each edge of the lattice with probability p such that self-connections and duplicate edges are excluded. This process introduces $pNK/2$ long-range edges which connect nodes that otherwise would be part of different neighborhoods. By varying p one can closely monitor the transition between order $(p = 0)$ and randomness $(p = 1)$.

Method on Generating a BA model

To generate an assigned scale of BA model, it must have the given parameters as inputs as seen in Table. 1.

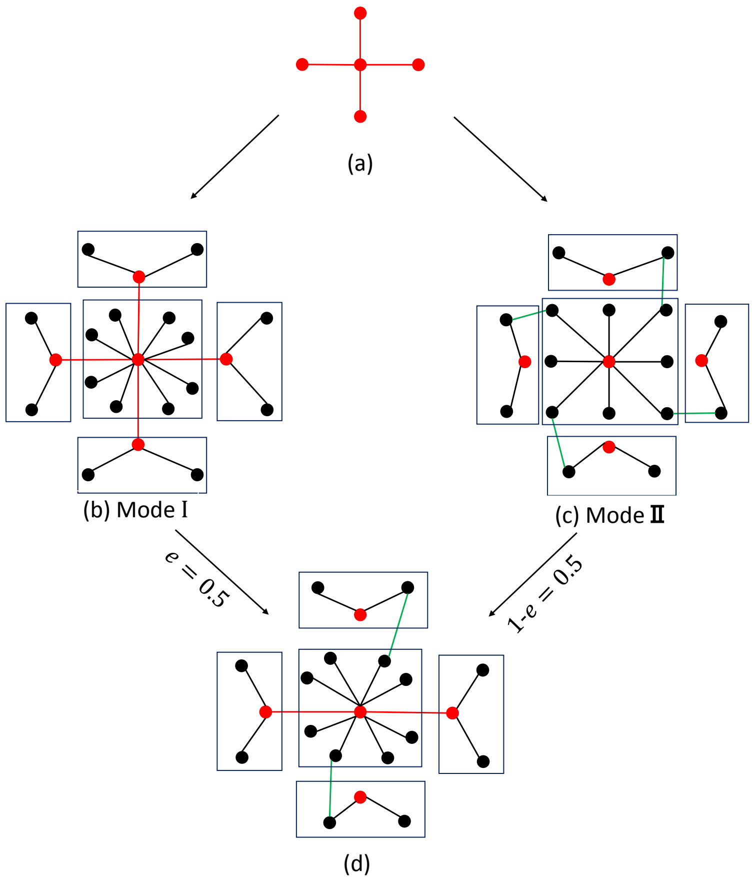


Figure 4: The minimal model [2]. Figure(a) is the initial structure at time step $t = 0$ and figure(b)(c)(d) are non-fractal, totally fractal, and with 50% fractality at time step $t = 1$, respectively.

Table 1: Given parameters for generating a BA model

Inputs	The number of initial connected nodes	The total number of nodes	The total number of links	The number of links added to a new node
	n	N	L	m

Based on these given parameters, we can obtain a probability, P, which can lead to the BA model generated as an assigned number of links.

$$L_0 = \frac{1}{2}n(n-1) + m(N-n) \tag{21}$$

$$P = \frac{L - L_0}{N - n} \tag{22}$$

There is a preferential attachment [11] of BA modelthat a new node trends to choose the existing node i to connect with the probability p_i. In the equation (23), k_i represents the degree of the existing node i, and the denominator of this equation represents the sum value of the degrees of all the existing nodes.

$$p_i = \frac{k_i}{\sum_j k_j} \tag{23}$$

Then the generating steps are as follows.

Step 1 Build an initial topology with n nodes to a completed graph.

Step 2 According to preferential attachment, choose an existing node.

Step 3 Add a new node to the chosen existing node.

Step 4 Get a temperate random value, r. If $r <= P$, then choose another m existing nodes for a new node to be added to. Else, choose another $(m-1)$ existing nodes for a new node to be added to.

Step 5 Return to Step 3 to add more new nodes until the total number of generated topology reaches to N.

Method on Generating a ER Model

To generate an assigned scale of random network by the method of ER model [15], it must have the given parameters as inputs as seen in Table. 2.

Based on these given parameters, we can obtain a probability, P, which can lead to the ER model model generated as an assigned number of links.

$$P = \frac{L}{\frac{1}{2}N(N-1)} \tag{24}$$

Then the generating steps are as follows.

Table 2: Given parameters for generating a ER model

Inputs	The total number of nodes	The total number of links
	N	L

Step 1 Get a new random value, ”ratio”, which is generated from Random Class for a node pair.

Step 2 If $ratio <= P$, add a link into this node pair.

Step 3 Return to step1 until each different node pairs have been given one and only one random value for adding a link.

Step 4 Check whether the final ER model is connected.

Step 4.1 If not, discard the final ER model and generate the ER model from beginning step1.

Step 4.2 Else, return a topology of this final ER model.

Method on Generating Lattice Graph models

The characteristic of lattice graph is that once the dimension and the number of nodes in each dimensional direction are determined, the scale of the number of nodes and the number of links are determined.

- 1-Dimensional Lattice Graph: Ring

The generating method of the ring topology is to make each node connect to its left neighbor node and its right neighbor node.

- 2-Dimensional Lattice Graph: Grid

The generating method of the grid is to make each node connect to its upper node, below node, left node and right node if its neighbor node exists.

The given parameters are in Table. 3.

Then the generating steps are as follows.

Table 3: Given parameters for generating a 2-dimensional lattice graph: Grid

Inputs	The number of nodes on x-axis	The number of nodes on y-axis
	N_x	N_y

Table 4: Given parameters for generating a 3-dimensional lattice graph: Cube

Inputs	The number of nodes on x-axis	The number of nodes on y-axis	The number of nodes on z-axis
	N_x	N_y	N_z

Step 1 Set a 2-dimensional coordinate (x_i, y_j), $0 \leq i < N_x, 0 \leq j < N_y$ to each node.

Step 2 Connect this node (x_i, y_j) to its upper, below, right and left nodes coordinate $(x_i - 1, y_j)$, $(x_i + 1, y_j)$, $(x_i, y_j - 1)$, $(x_i, y_j + 1)$. But if the coordinate is over the coordinate defined range, do not connect.

- 3-Dimensional Lattice Graph: Cube

The generating method of the cube is to make each node connect to its upper node, below node, left node, right node, front node and behind node, if its neighbor node exists.

The given parameters are in Table. 4.

Then the generating steps are as follows.

Step 1 Set a 2-dimensional coordinate (x_i, y_j, z_k), $0 \leq i < N_x, 0 \leq j < N_y, 0 \leq k < N_z$ to each node.

Step 2 Connect this node (x_i, y_j) to its upper, below, right and left nodes coordinate $(x_i - 1, y_j, z_k)$, $(x_i + 1, y_j, z_k)$, $(x_i, y_j - 1, z_k)$, $(x_i, y_j + 1, z_k)$, $(x_i, y_j, z_k - 1)$, $(x_i, y_j, z_k + 1)$. But if the coordinate is over the coordinate defined range, do not connect.

5.1.2 Condition Settings

I have introduced four aspects of condition settings for this evaluation.

Table 5: The supporting combination relation between virtual networks and physical topology candidates in this study.

Physical Topology Candidates	Fractal virtual network	FullMesh virtual network
	Hub failure	Random node failure
Lattice1D-Ring	○	○
Lattice2D-Grid	○	○
Lattice3D-Cube	○	○
Scale Free-BAmodel	○	○
Small World-WSmodel	○	○
Random Graph-ERmodel	○	○
Fractal	○	○
Star	×	×

First, node matching from virtual network to physical topology is necessary and we match them at random. In the real world, virtual networks are constructed after the physical topology exists so that what nodes of physical topology are used as the nodes of virtual networks has been determined as virtual networks appear. However, we design a physical topology in this study for virtual networks which will be widely used in the next generation information networks, and this makes that there is no corresponding relation between physical topology nodes and virtual network nodes in simulations.

Random node matching can simulate all kinds of cases in the real world. Even setting random matching can relax the traffic concentrating on hubs, comparing to node matching based on nodal degree. However, to obtain the general results, we will simulate the physical candidates by many random numbers and use the average data as the results on average performances.

Second, to prove the result of this study which can support fractal virtual networks well, the evaluation on supporting a full mesh virtual network as a contrast is necessary. That is to say that we will evaluate physical topology candidates both on supporting fractal virtual networks and on a supporting full mesh virtual network.

Third, we simulate a node failure situation for virtual networks and observe how physical topology candidates perform on cost for supporting virtual networks. In the real world, node failures usually happen in the physical topology, and the corresponding nodes in virtual networks are also failed at the same time as virtual nodes use the resources of the physical topology. Considering about this reality, we set the situation for this study that failed nodes found in virtual networks means that the corresponding nodes in physical topology candidates are also out of work. In other words, we will simulate the node failure in both virtual networks and physical topology candidates, but the type of node failure is determined by virtual networks and physical topology candidates will meet node failures following the failed nodes in virtual networks.

Moreover, we will simulate the type of hub (the node with the largest degree) failures in fractal virtual networks and the type of random node failures in full mesh virtual networks. Fractal networks have a highly prominent robustness against hub failures. To set the situation of hub failures in fractal virtual networks is because we expect that our designed physical topology can not only support fractal virtual networks with low costs when they function normally but also can support them robustly both on connectivity and on low costs when they meet hub failures which are the advanced aspect of fractal virtual networks.

And we set a situation for supporting full mesh virtual networks which will meet with random node failures is because all nodes in a full mesh network have the same degree. In other words, in full mesh networks, such homogeneous networks, we can only use a random method to simulate node failures.

At last, we use the average performances of each physical topology as the final evaluation results. We do the evaluation by three steps which are generating topology models, nodes matching and evaluating under node failures. Random numbers are used at each steps to generate a model with assigned scale and to match nodes at random and to remove failed nodes at random, respectively. To avoid the effect of randomness on evaluation results, we calculate the average evaluation value for each physical topology candidate. Therefore, all the evaluation results in this thesis represent the average performances which have the generality in information networks.

5.1.3 Evaluating Measures

According to the formula (10) with constraints (12)–(17), there are two main measures for this study to be focused on in the numerical evaluation, which are “cost” and “relative cost”. I prefer to make a detailed description on definitions of both cost and relative cost again in this numerical evaluation though they have been defined on the Section 3 of problem formulation.

"Cost" refers to the total distances which physical topology candidate takes to support all accessible node pairs in the virtual network, and it measures on flexibility, how much average distance cost the physical topology takes. The accessible node pair is a node pair which has, at least, one path between them on graph theory. The distance is the length of the shortest path of an accessible node pair. In this study, we simulate the networks on a simple condition that network models are undirected and unweighted on graph theory, also called "binary graph". Therefore, it is no need to consider about the information transferring direction between each accessible node pair, and the distance is the smallest number of hops between each accessible node pair. It is noteworthy that we only consider the accessible node pairs in virtual networks and calculate the distances of corresponding node pairs in the physical topology as cost. A virtual network may lose connectivity which means that not all nodes have a path for transferred information to be able to arrival at all other nodes, and physical topology only supports its virtual network at the functioned part.

"Relative cost" refers to the rate of cost changing compared cost to correlative original cost, and it represents a measure of robustness, how much the cost changes with holding the connectivity under hub failures. Correlative original cost refers to the cost of the currently functioned part on the original state of both physical topology and virtual network, which is a normal state without any failure in this study.

5.2 Results of Numerical Evaluations

We obtained the results of the numerical evaluation of following two situations, which are to support a fractal virtual network with hub failures and to support a full mesh virtual network with random node failures, respectively. The scale of each topology model can be seen as Table 7 and Table. tab:basic of vn, and the degree distribution of each topology models can be seen as Fig. 5 – Fig. 8. We do the evaluation and comparing based on the golden rule [16] that the graphs to be compared must have the same number of nodes and the same number of edges. Moreover, the effect of randomness will be eliminated as Table. 8

Table 6: Basic properties of two types of virtual networks in the evaluation.

Topology Type	Total number of nodes	Total number of links	Average degree
Fullmesh	100	4950	99
Fractal Network	100	218	4.36

Table 7: Basic properties of topologies used in this evaluation. Star is not used as a candidate, because its poor performance is obvious.

Topology Type	Total number of nodes	Total number of links	Average degree
Lattice1D-Ring	100	100	2
Lattice2D-Grid	100	180	3.6
Lattice3D-Cube	100	235	4.7
Scale Free-BAmodel	100	235	4.7
Small World-WSmodel	100	235	4.7
Random Graph-ERmodel	100	235	4.7
Fractal Topology	100	218	4.36
(Star)	(100)	(99)	(1.98)

5.2.1 When Physical Topology Candidates Support a Fractal Virtual Network

According to Fig. 9 and Fig. 10, we can obtain the results from figures as follows. And based on these results, we obtain a conclusion that "ScaleFree-BAmodel" is optimal to support a fractal virtual network among these physical topology candidates, because it has the lowest cost with the stable changing and can hold the connectivity under hub failure.

On Origin Cost "ScaleFree-BAmodel" is optimal. (as seen in the upper panel of Fig. 9)

ScaleFree-BAmodel $<$ RandomGraph-ERmodel $<$ SmallWorld-WSmodel $<$ Lattice3D-Cube $<$ Fractal Topology $<$ Lattice2D-Grid $<$ Lattice1D-Ring

On Cost under Hub Failure "ScaleFree-BAmodel" is optimal. (as seen in Fig. 9)

Random Graph-ERmodel $<$ SmallWorld-WSmodel $<$ Lattice3D-Cube $<$ Fractal Topology $<$ Lattice2D-Grid $<$ Lattice1D-Ring

Table 8: To obtain the average performance, the number of avoiding the effects of randomness for each physical topology candidate is shown. Each Lattice graph only have one pattern under 100 nodes scale.

Physical Topology	Total number of samples of each PT	Total number of eliminating other randomness	Total number for average performance
Lattice1D-Ring	1	10	10
Lattice2D-Grid	1	10	10
Lattice3D-Cube	1	10	10
Scale Free-BAmodel	10	10	100
Small World-WSmodel	10	10	100
Random Graph	10	10	100
Fractal	5	10	50

On Connectivity against Hub Failure Except ”Lattice1D-Ring”, all other can hold the connectivity until 0.04 rate of hub failures in the fractal virtual network. (as seen in Fig. 9)

On Cost Changing under Hub Failure “Lattice3D-Cube” is the most stable. (as seen in Fig. 10)

Lattice3D-Cube $<$ ScaleFree-BAmodel $<$ Lattice2D-Grid $<$ Random Graph-ERmodel $<$ SmallWorld-WSmodel $<$ Fractal Topology $<$ Lattice1D-Ring

First, I generate a fractal model [2, 3] as the fractal virtual network in this numerical evaluation. The fractal model will lose its connectivity which means unconnected network on graph theory when it meets with four hub failures. When physical topology candidates support such a failed fractal virtual network, the cost will get fewer and fewer because the number of accessible node pairs which need to be supported is getting smaller as the number of failed hubs increases. However, the measure of relative cost can observe the changing of cost on the same node pair. More details will be introduced at following data figures. Moreover, physical topology candidates are evaluated only under the states that virtual networks are connected.

On the other hand, physical topology candidates have no much differences on connectivity under failures, expressed by the length as seen in Fig. 9 in this simulation. It is because the fractal virtual network in the simulation can hold the connectivity until 0.04 hub failures which damage the network seriously, while physical topology candidates can almost keep connected with this low rate of random failures.

However, it is noteworthy on the robustness of connectivity of each physical topology candidate. The link scales are different which can affect the results on connectivity, although we compared the connectivity and derive the results from data. As seen in Table. 7, candidates of Lattice1D-Ring, Lattice2D-Grid and Fractal Topology are different from others since the characteristics on structures lead to the fixed average degrees.

Cost represents that the lower the cost of a candidate is, the more efficient the candidate can support the fractal virtual network both on the normal state and on the hub failures condition.

Fig. 9 shows the cost of each physical topology candidate supporting all of the accessible node pairs when hubs failed one by one in the fractal virtual network, and different ranges of the vertical axis make the distinguishes among candidates more visualized.

It is obvious to find that all of these costs decrease as more hubs failed. The reason why costs get fewer after there is node failure in networks is that accessible node pairs to be supported get fewer as more and more nodes fail. The number decreasing of accessible node pairs to be supported is because the node scale gets smaller due to failed nodes and the fractal virtual network lose its connectivity which leads to accessible node pairs much fewer.

As comparing costs is under the condition that all candidates with same network scale support the same node pairs, data of candidates after they lose connectivity cannot be compared to others which hold their connectivity. In Fig. 9 (a), the plots on the vertical axis represent the costs in the normal state of virtual network and the lower the cost is , the more flexible the physical topology candidate is. In Fig. 9 (b), curses represent the robustness against failures, and the lower and longer the curve is, the more robust the physical topology candidate is.

Therefore, we can obtain some results from Fig. 9 as follows.

- When the fractal virtual network is normal without failures, the relation between costs of candidates,
  "ScaleFree-BAmodel $<$ RandomGraph-ERmodel $<$ SmallWorld-WSmodel $<$ Lattice3D-Cube $<$ Fractal topology $<$ Lattice2D-Grid $<$ Lattice1D-Ring",

which means the efficiency of each candidate is from high to low in the order following this relation for supporting the fractal virtual network under the normal state.

- After the fractal virtual network meets with hub failures, all of candidates can hold the connectivity somehow, and have the cost relation,
  "ScaleFree-BAmodel $<$ RandomGraph-ERmodel $<$ SmallWorld-WSmodel $<$ Lattice3D-Cube $<$ Fractal topology $<$ Lattice2D-Grid $<$ Lattice1D-Ring",
  which means the efficiency of each candidate is from high to low in the order following this relation for supporting the fractal virtual network under the hub failures condition.

- As Lattice2D-Grid, Lattice1D-Ring and Fractal Topology have different network scales from other candidates, they cannot compare their costs to others, but their costs are really high which means low efficiency.

On the other aspect, the relative cost can express that smaller the relative cost is, the more robust the candidate supports the fractal virtual network against hub failures in virtual networks.

Fig. 10 shows the relative cost of each physical topology candidate supporting all of the accessible node pairs when hubs failed one by one in the fractal virtual network, and different ranges of the vertical axis make the distinguishes among candidates more visualized.

As relative costs show the cost changing from the normal condition for the same supporting node pairs, the value will be larger than 1.

Here, we also need to the comparing premise of what candidates are under the same condition. Only the relative costs of RandomGraph-ERmodel, SmallWorld-WSmodel and Lattice3D-Cube can be compared when the fractal virtual network meets with hub failures, it is because they have the same network scale and can hold the connectivity somehow. Therefore, we can obtain some results from Fig. 10 as follows.

- Under holding the connectivity, the relative cost which represents the cost changing comparing to the origin cost of the same supporting node pairs shows that how slowly the cost gets high has the relation,
  "Lattice3D-Cube $<$ ScaleFree-BAmode $<$ Lattice2D-Grid $<$ Random Graph-ERmodel $<$ SmallWorld-WSmodel $<$ Fractal Topology $<$ Lattice1D-Ring",

which means that less cost changing is, more robust the candidate supports the fractal virtual network against hub failures.

- Lattice3D-Cube has a little or no cost changing which means hub failures in the fractal virtual network bring less change on the cost of Lattice3D-Cube. However, it has high cost so that the least changing on cost under failures means to remain the high cost. Thus, Lattice3D-Cube with the least changing on cost is not optimal.
- Except Lattice3D-Cube, the least relative cost is from ScaleFree-BAmodel under failures. According to its lowest cost in normal state, ScaleFree-BAmodel can keep the lowest cost all the times.
- The effect from the different scales should be considered on comparing, which are Fractal Topology, Lattice1D-Ring and Lattice2D-Grid.

Therefore, based on all above illustration and analysis of data, we obtain that "ScaleFree-BAmodel" is optimal to support a fractal virtual network among these physical topology candidates, because it has the lowest cost, which is the most flexible, with the most stable changing on low cost and can hold the connectivity under hub failure, which is the most robust.

5.2.2 When Physical Topology Candidates Support a Full Mesh Virtual Network

According to Fig. 11 and Fig. 12, we can obtain the results from figures as follows. And based on these results, "ScaleFree-BAmodel" is optimal to support a full mesh virtual network, because it has the lowest cost with the second most stable changing and can hold the connectivity under random node failure.

On Origin Cost "ScaleFree-BAmodel" is optimal. (as seen in Fig. 11 (a))

ScaleFree-BAmodel < Random Graph-ERmodel < SmallWorld-WSmodel < Lattice3D-Cube < Fractal Network < Lattice2D-Grid < Lattice1D-Ring

On Cost under Random Node Failure "ScaleFree-BAmodel" is optimal. (as seen in Fig. 11 (b))

ScaleFree-BAmodel < Random Graph-ERmodel < SmallWorld-WSmodel < Lattice3D-Cube < Fractal Network < Lattice2D-Grid < Lattice1D-Ring

Table 9: Basic properties of physical topology candidates for supporting the fullmesh virtual network on average performances when meeting with random node failures until 0.2.

Physical Topology Candidate	The number of failed nodes once it loses connectivity (and also cannot support VN)	The ratio of failed nodes once it loses connectivity (and also cannot support VN)
Fractal	14	0.14
Lattice1D-Ring	1	0.01
Lattice2D-Grid	15	0.15
Lattice3D-Cube	18	0.18
Scale Free-BAmodel	14	0.14
Small World-WSmodel	18	0.18
Random Graph	13	0.13

On Connectivity against Random Node Failure All candidates can somehow hold the connectivity with 0.2 rate of random node failures, except Lattice1D-Ring.

On Cost Changing under Hub Failure "Lattice3D-Cube" is the most stable. (as seen in Fig. 12)

Lattice3D-Cube $<$ ScaleFree-BAmodel $<$ Lattice2D-Grid $<$ Random Graph-ERmodel $<$ SmallWorld-WSmodel $<$ Fractal Topology $<$ Lattice1D-Ring

Because the full mesh is a completed graph in which each node can transfer information to any other nodes, we only list the basic properties of physical topology candidates for supporting the full mesh virtual network (seen as Table 9). And there is no doubt that when physical topology loses connectivity is the time when it cannot support the full mesh virtual network anymore.

Particularly, the data in Table 9 are the average connectivity of each candidates within 0.2 rate of random node failures in virtual network and physical topologies. We eliminate the effect of randomness on the evaluation results to evaluate many samples with different random numbers and calculate the average value of these samples for each type of candidate as the average performance.

As the meanings of both cost and relative cost have been explained in the above part at this section and all plots are shown when their candidates hold the connectivity, we can derived some results from Fig. 11 and Fig. 12 directly.

In Fig. 11, the reason of the cost decreasing as failed nodes increase indicates is only that the node scale gets smaller due to the failed nodes. We can derive some results from Fig. 11 as follows.

- When the full mesh virtual network is under the normal state, the relation between cost of each candidate has,
  "ScaleFree-BAmode $<$ RandomGraph-ERmodel $<$ SmallWorld-WSmodel $<$ Lattice3D-Cube $<$ Fractal Network $<$ Lattice2D-Grid $<$ Lattice1D-Ring" ,
  which indicates that the efficiency of each candidate is from high to low in the order following this relation for supporting the normal full mesh virtual network .

- When the full mesh virtual network meets with random node failures, the relation between cost of each candidate has,
  "ScaleFree-BAmode $<$ RandomGraph-ERmodel $<$ SmallWorld-WSmodel $<$ Lattice3D-Cube $<$ Fractal Network $<$ Lattice2D-Grid $<$ Lattice1D-Ring" ,
  which indicates that the efficiency of each candidate is from high to low in the order following this relation for supporting the full mesh virtual network with random node failures.

- The different link scales should been considered when compareing Lattice1D-Ring, Lattice2D-Grid and Fractal Topology to others.

On the other aspect, we can derive some results from Fig. 12 as follows. The randomness has been eliminated.

- When the full mesh virtual network meets with random node failures, before the ratio of failed nodes increases to 0.2, the relation between relative cost of each candidate has,
  "Lattice3D-Cube $<$ ScaleFree-BAmodel $<$ Lattice2D-Grid $<$ RandomGraph-ERmodel $<$ SmallWorld-WSmodel $<$ Fractal Topology $<$ Lattice1D-Ring",
  which means that smaller the relative cost is, more robust the candidate supports the full mesh virtual network against random node failures.

- Lattice3D-Cube has a little or no cost changing which means hub failures in the fullmesh virtual network bring less change on the cost of Lattice3D-Cube. However, it has high cost which leads to the least changing on cost under failures means to remain the high cost. Thus, Lattice3D-Cube with the least changing on cost is not optimal.

- Except Lattice3D-Cube, the least relative cost is from ScaleFree-BAmodel under failures. According to its lowest cost in normal state, ScaleFree-BAmodel can keep the lowest cost all the times.
- The different link scales should been considered when compareing Lattice1D-Ring, Lattice2D-Grid and Fractal Topology to others.

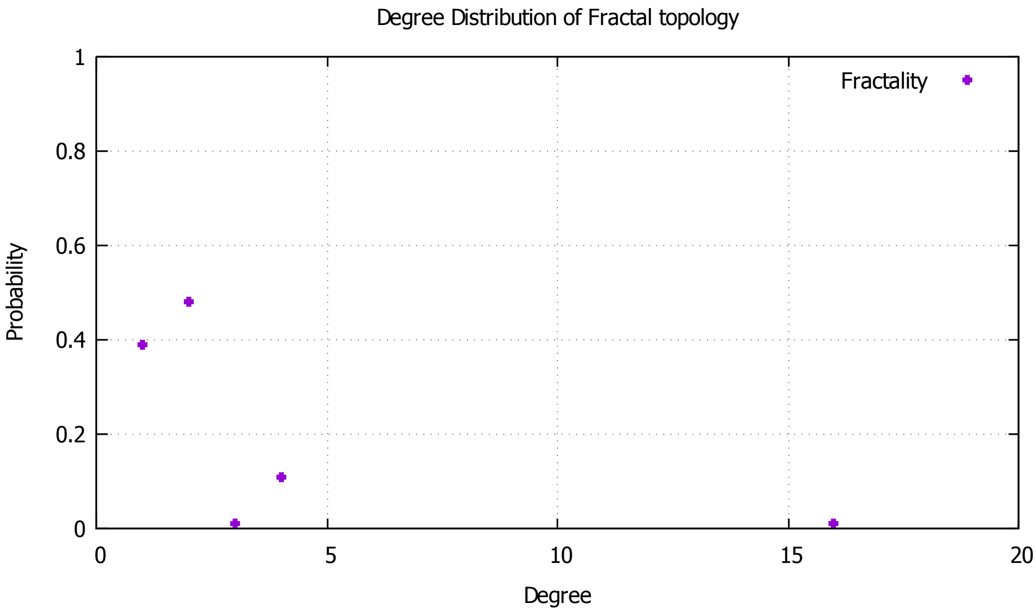
Therefore, based on all above illustration and analysis of data, we obtain that “ScaleFree-BAmodel” is optimal to support a full mesh virtual network, because it has the lowest cost, which is the most flexible, with the most stable changing on the lowest cost and can hold the connectivity under random node failure, which is the most robust.

5.2.3 Comparing the Situation of Supporting A Full Mesh Virtual Network to Both the Situation of Supporting A Fractal Virtual Network and Brain Networks

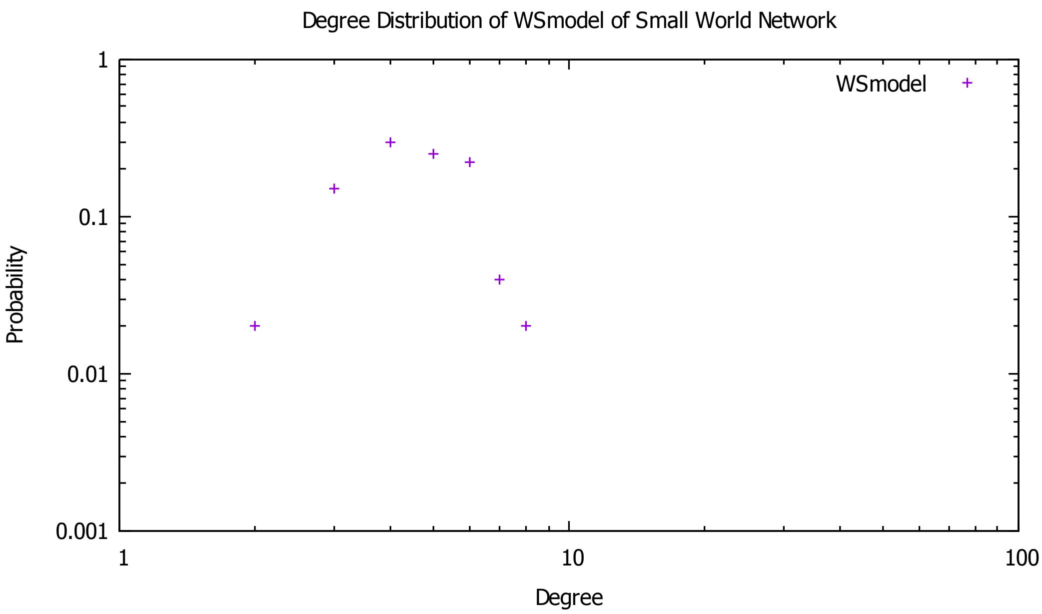
We set the situation of supporting a full mesh virtual network to simulate a brain network environment. Although the existing of edges in brain functional networks is determined by a set threshold, the functional networks are regarded as a full mesh generally. Since there is no samples or appreciate samples of brain structural networks, we try to infer how the result of the structural networks will be by this evaluation. Under the requirement in information networks, high efficiency is expected. Thus, we regard brain structural networks as a state of providing the highest efficiency to brain functional networks. In this case, we infer that the result of brain structural networks will locate below all the curves to reach the lowest cost and smallest changing rate of distance cost.

On the other side, comparing the situation of supporting the fullmesh virtual networks to the situation of supporting a fractal virtual network, we obtain that the results are same to each other. However, we have not known the effect of average nodal degree on scale on the evaluation results, even the result [1] that the random topology is the most efficient in brain networks.

It indicates that the result of physical topology representing brain structural networks cannot be applied directly in information networks as a physical topology to support fractal virtual network. Therefore, it is necessary to find or design a physical topology to support fractal virtual network flexibly and robustly. Furthermore, we find the physical topology with a topological feature of random graphs is optimal to support fractal virtual network flexibly and robustly.

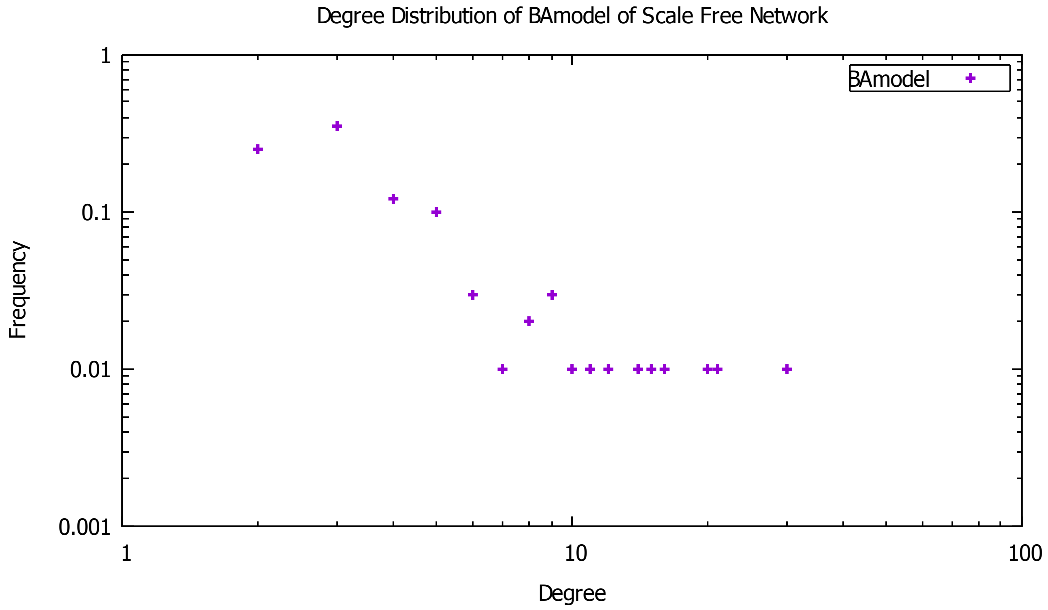


(a) Fractal topology

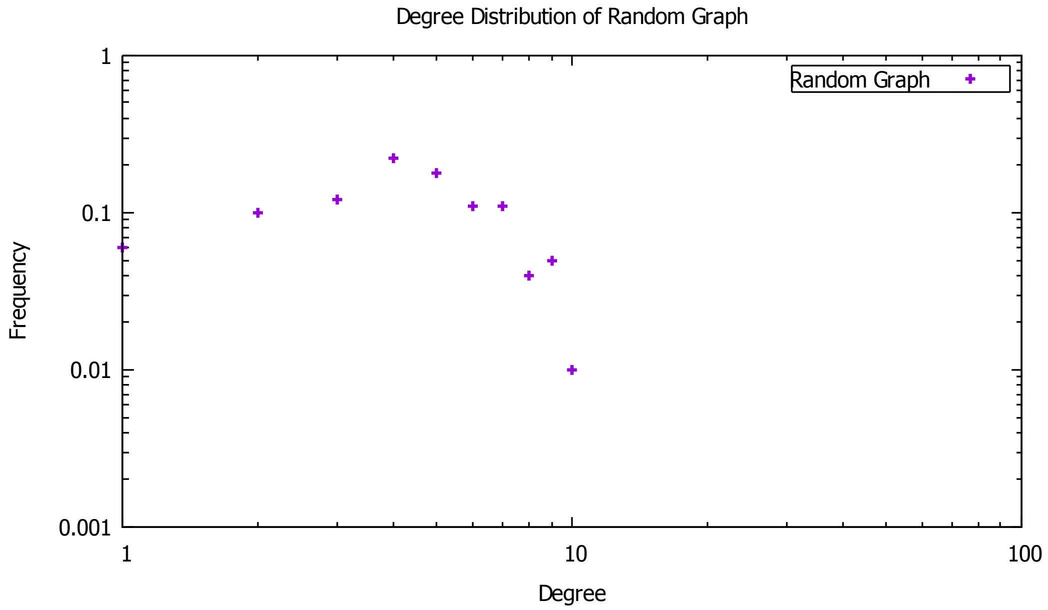


(b) Small World - WS model

Figure 5: Degree Distributions (1)

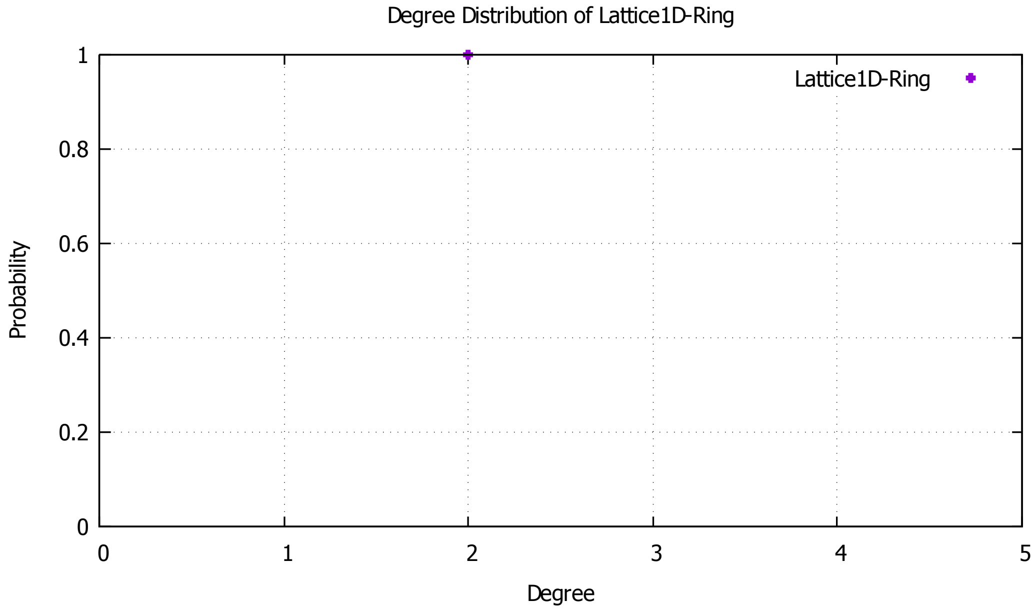


(a) Scale Free - BA model

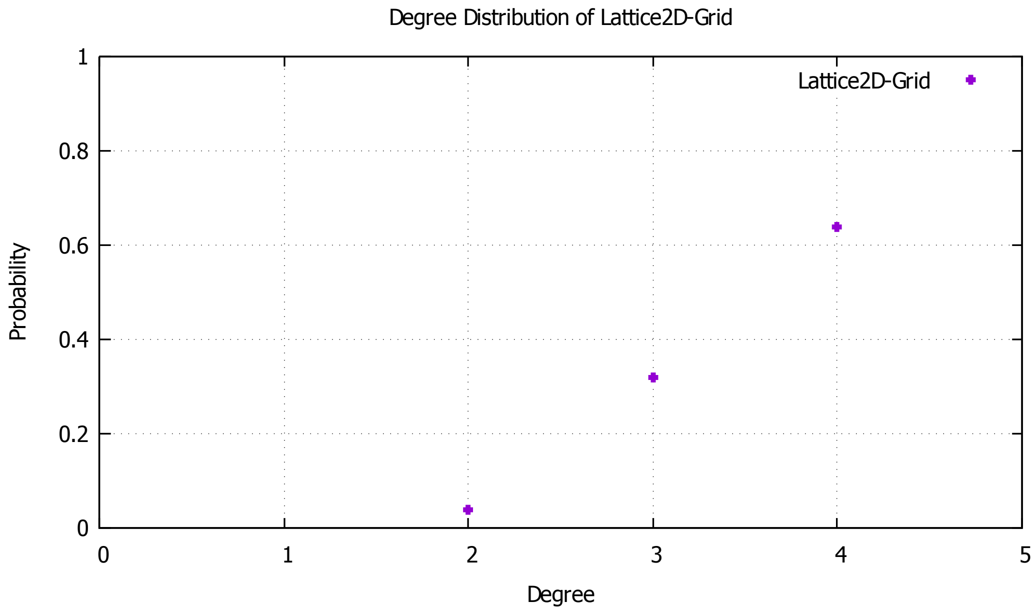


(b) Random Graph - ER model

Figure 6: Degree Distributions (2)



(a) Lattice1D - Ring



(b) Lattice2D - Grid

Figure 7: Degree Distributions (3)

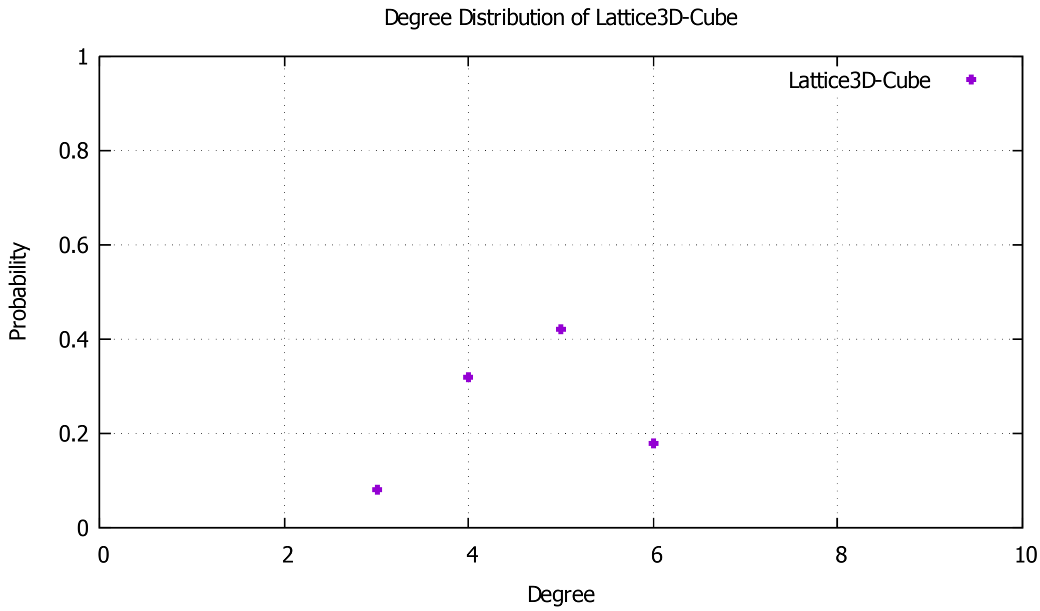
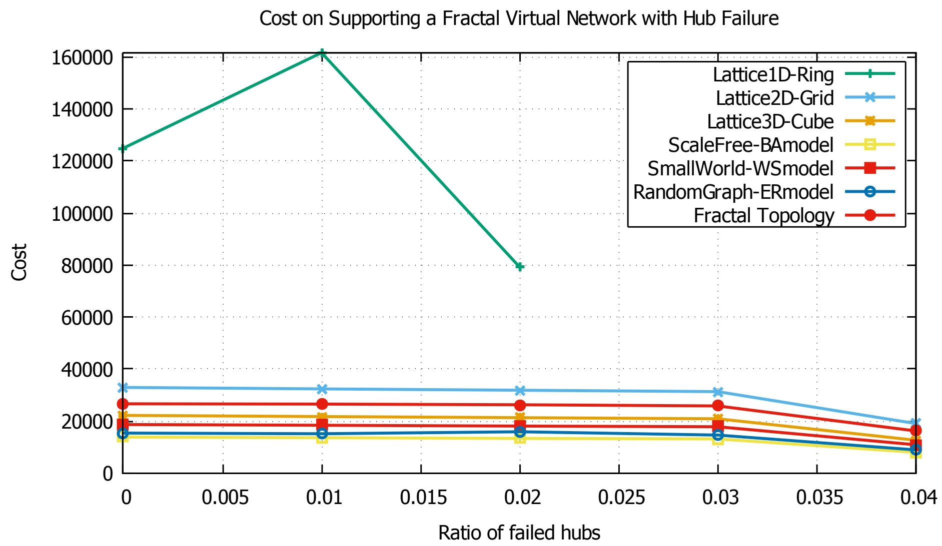
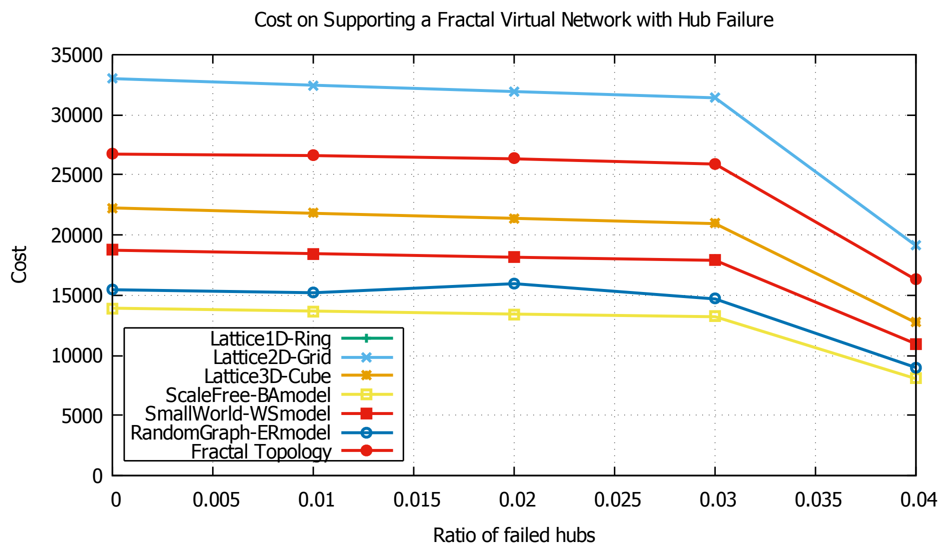


Figure 8: Degree Distribution (4)

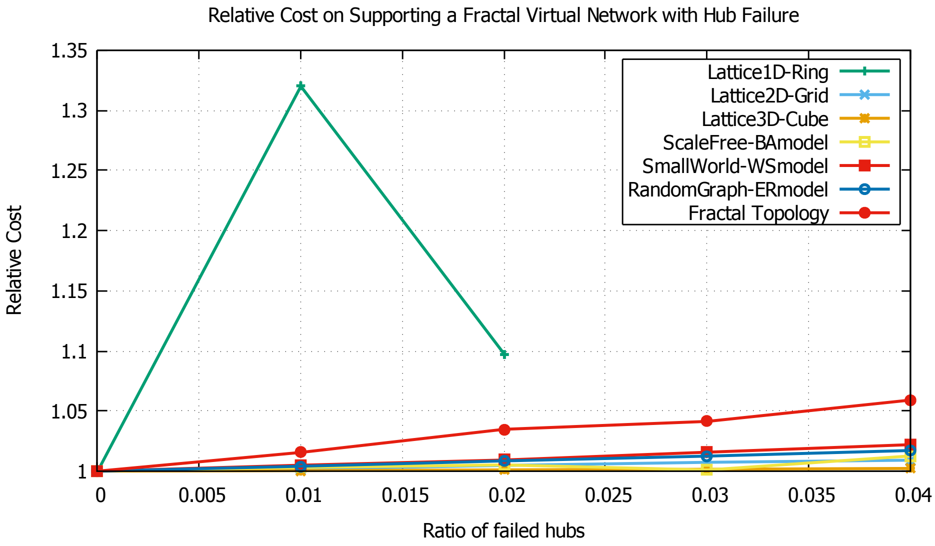


(a) Figure shows the whole cost performance of each candidate with 0-161,700 range of the vertical axis. And the costs on the normal state can be seen.

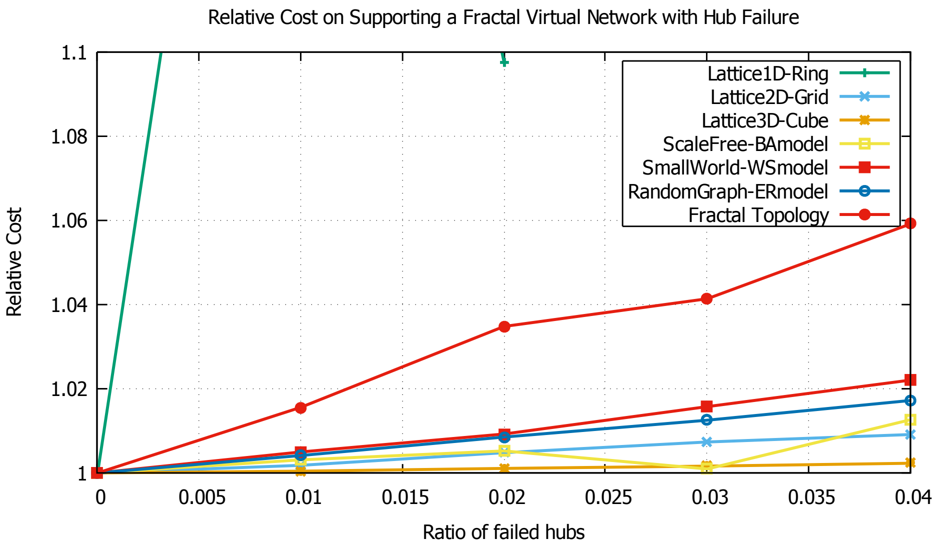


(b) Figure with 0-35,000 range of the vertical axis shows the candidate costs of 0-0.04 ratio of failed hubs more clearly, and mainly shows the costs comparing among BAmodel, WSmodel, Cube and ERmodel as they hold the connectivity and have the same scale.

Figure 9: Cost of each physical topology candidate on supporting the fractal virtual network. All plots are shown when candidates hold their connectivity on average performances.

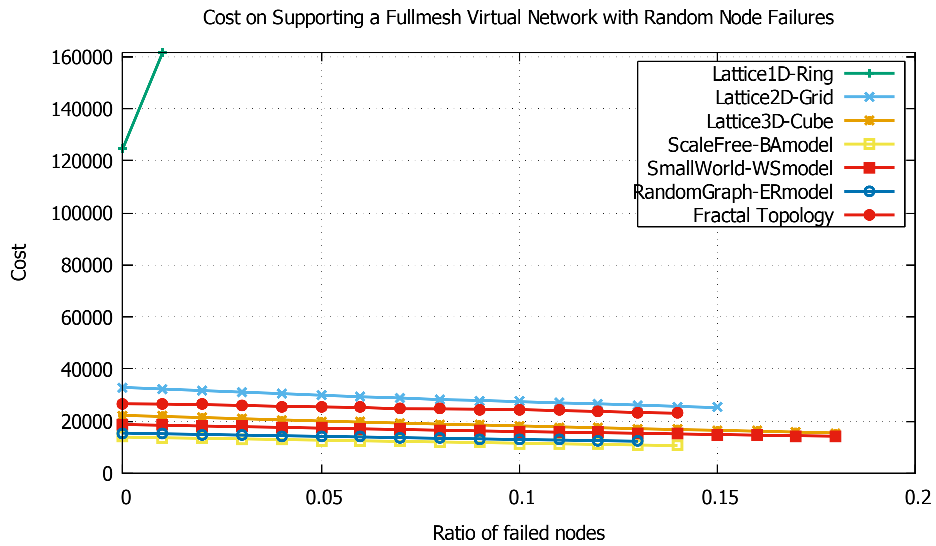


(a) Figure with 1-1.3 range of the vertical axis shows the whole relative cost performance of each candidate.

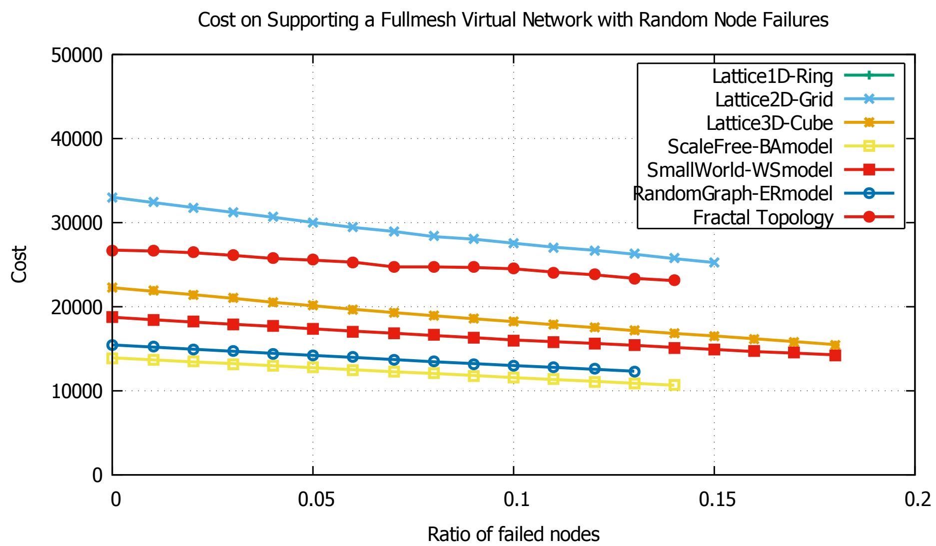


(b) Figure with 1-1.1 range of the vertical axis shows the distinguishes among relative cost of each candidate more clearly.

Figure 10: Relative cost of each physical topology candidate on supporting the fractal virtual network. All plots are shown when candidates hold their connectivity on average performances.



(a) Figure with 0-161,700 range of the vertical axis shows the whole cost performance of each candidate.



(b) Figure with 0-50,000 range of the vertical axis shows the candidate costs more clearly.

Figure 11: Cost of each physical topology candidate on supporting the fuilmesh virtual network. All plots are shown when candidates hold their connectivity on average performances.

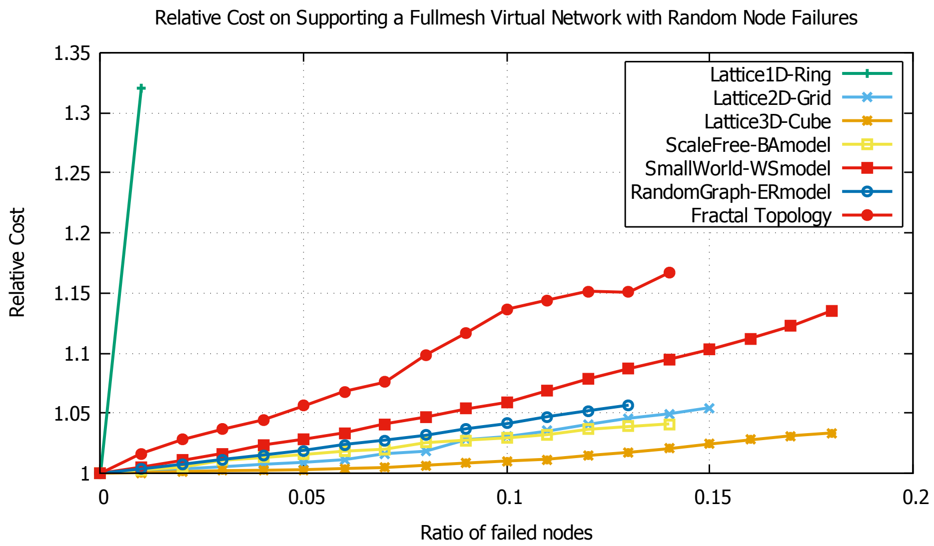


Figure 12: Relative cost of each physical topology candidate on supporting the fractal virtual network. All plots are shown when candidates hold their connectivity on average performances.

6 Conclusion

We investigated the mechanism of how structural connectivity supporting functional connectivity in brain networks and we were inspired that we should find the optimal topological property for physical topology to support fractal virtual networks flexibly with low distance cost and robustly on holding the connectivity with less distance cost changing against hub failures. Based on these requirements, we formulated the optimization problem of physical topology with capacity and connectivity requirements which can provide an approach on investigating the optimal physical topology. However, the optimal physical topology cannot be directly derived by following the general solution of the optimal problem. We proposed an mathematical formulation for solving the optimal problem to decide physical topology. We enumerated many types of topological properties and generated topology models with these topological properties as the physical topology candidates for numerical evaluation on investigating the optimal topological property. We applied our mathematical formula on numerical evaluation to evaluate the average performances. We found that the scale-free physical topology where the degree distribution follows a power law could support fractal virtual networks flexibly and robustly. We also simulated a brain network case in which a full mesh virtual network was as the brain functional connectivity. We found that the result was same as the case of supporting fractal virtual networks,but different from the results in brain networks. And the result indicates that the topology of brain structural connectivity cannot directly apply to information networks to support fractal virtual networks. Therefore, it is necessary to find a physical topology as our results to support fractal virtual networks optimally.

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References

[1] E. Bullmore and O. Sporns, “The economy of brain network organization,” *Nature Reviews Neuroscience*, vol. 13, pp. 336–349, May 2012.

[2] C. Song, S. Haclin, and H. A. Makse, “Origins of fractality in the growth of complex networks,” *Nature physics*, vol. 2, pp. 275–281, Apr. 2006.

[3] Y. Shijo, “A configuration method of virtual networks with hierarchical robustness inspired by the fractality of brain functional networks,” Master’s thesis, Osaka University, Feb. 2016.

[4] E. T.Bullmore and D. S. Bassett, “Brain graphs: Graphical models of the human brain connectome,” *Annual review of clinical psychology*, vol. 7, pp. 113–140, 2011.

[5] C. J. Honey, O. Sporns, L. Cammoun, X. Gigandet, J.-P. Thiran, R. Meuli, and P. Hagmann, “Predicting human resting-state functional connectivity from structural connectivity,” *Proceedings of the National Academy of Sciences*, vol. 106, pp. 2035–2040, Feb. 2009.

[6] S. Sarkar, S. Chawla, and D. Xu, “On inferring structural connectivity from brain functional-mri data,” *arXiv preprint arXiv:1502.06659*, Feb. 2015.

[7] C.-Y. Lo, P.-N. Wang, K.-H. Chou, J. Wang, Y. He, and C.-P. Lin, “Diffusion tensor tractography reveals abnormal topological organization in structural cortical networks in alzheimer’s disease,” *The Journal of Neuroscience*, vol. 30, pp. 16876–16885, Dec. 2010.

[8] R. L. Buckner, J. Sepulcre, T. Talukdar, F. M. Krienen, H. Liu, T. Hedden, J. R. Andrews-Hanna, R. A. Sperling, and K. A. Johnson, “Cortical hubs revealed by intrinsic functional connectivity: mapping, assessment of stability, and relation to alzheimer’s disease,” *The Journal of Neuroscience*, vol. 29, pp. 1860–1873, Feb. 2009.

[9] D. S. Bassett and E. T. Bullmore, “Human brain networks in health and disease,” *Curr Opin Neurol*, vol. 22, pp. 340–347, Aug. 2009.

[10] D. J. Watts and S. H. Strogatz, “Collective dynamics of s̀mall-world’ networks,” *nature*, vol. 393, pp. 440–442, June 1998.

[11] R. Albert and A.-L. Barabasi, “Statistical mechanics of complex networks,” *Reviews of modern physics*, vol. 74, pp. 47–97, Jan. 2002.

[12] C. Song, L. K. Gallos, S. Havlin, and H. A. Makse, “How to calculate the fractal dimension of a complex network: the box covering algorithm,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2007, p. P03006, Mar. 2007.

[13] C. M. Schneider, T. A. Kesserlring, J. S. A. Jr, and H. J. Herrmann, “Box-covering algorithm for fractal dimension of complex networks,” *Physical Review E*, vol. 86, p. 016707, July 2012.

[14] J. S. Kim, K. I. Goh, B. Kahng, and D. Kim, “A box-covering algorithm for fractal scaling in scale-free networks,” *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 17, June 2007.

[15] P. Erdos and A. Renyi, “On random graphs i,” *Publ. Math. Debrecen*, vol. 6, pp. 290–297, Nov. 1959.

[16] B. Bollobas, *Random Graphs*. Academic Press, 1985.