Quantum Decision Making with Small Sample for Network Monitoring and Control

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Network Monitoring

- With the development and diversification of applications on the Internet, applications that require high responsiveness, such as video streaming, are becoming mainstream.
- Application responsiveness is not only a matter of communication delay but also a matter of time required to grasp changes in network conditions.
- The **tradeoff between accuracy and measurement time** is a challenge in network control.

Decision Making with Small Sample

- When making decisions, **people are known to make appropriate choices based on relatively small samples**.
- Although there have been various studies on models of human decision-making, a model that integrates various cognitive biases, called "quantum decision making," has recently attracted much attention.
- However, the modeling of small samples has not been examined much so far.
- In this paper, **we extend the model of quantum decision making to model decision making with small sample**.

Decision Problem Setting

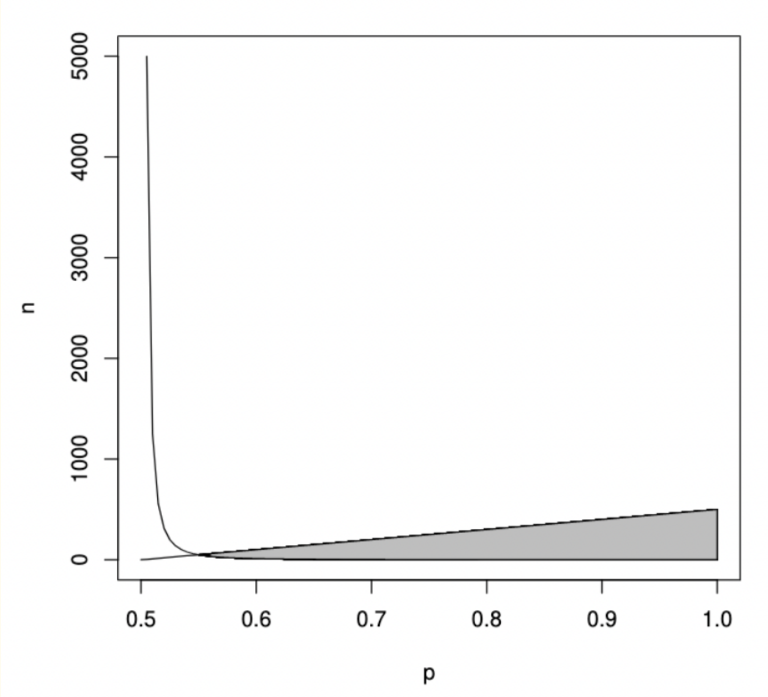
- Choosing option i out of k options yields a reward $x > 0$ with probability p_i and a reward $y = 0$ with probability $1 - p_i$ where $\sum_i p_i = 1$.
  - It also does not lose generality as $p_1 > p_2 > \cdots > p_k$.
  - The correct answer is selecting the choice $i = 1$.
- The decision-maker selects the choice with the highest probability estimated with the samples.

Quantum Decision Making with Samples

- Cognitive State
  - $|x\rangle$: k-dimensional state
  - x: One-hot vector
- Update State with Sample(Amplitude Amplification)
  - $|x_{t+1}\rangle = U_{D(\theta)} U_\omega |x_t\rangle$
    - $U_{D(\theta)} = 2|D(\theta)\rangle\langle D(\theta)| - I$
    - $U_\omega = I - 2|\omega\rangle\langle\omega|$
    - $|D(\theta)\rangle = \sin\theta\,|\omega\rangle + \cos\theta\,|\omega^\perp\rangle$
    - ω : target choice

Analytical Number of Samples

- Set of cases that the decision-maker chooses the best choice
  - $S = \{n_1 > n_i | i = 2, \cdots, k\}$
  - n_i: number of rewards with choice i
- Lower bound of $n = \sum_i n_i$ where $P(S) \geq \theta$ is estimated as follows.
  - $n \geq \frac{C_V(p_2)}{C_E(p_2)} \frac{1}{\theta^{\frac{1}{1-k}} - 1}$
  - $C_E(p) = p_1 - p, C_V(p_i) = p_1(1 - p_1) + p_i(1 - p_i) + 2p_1p_i$

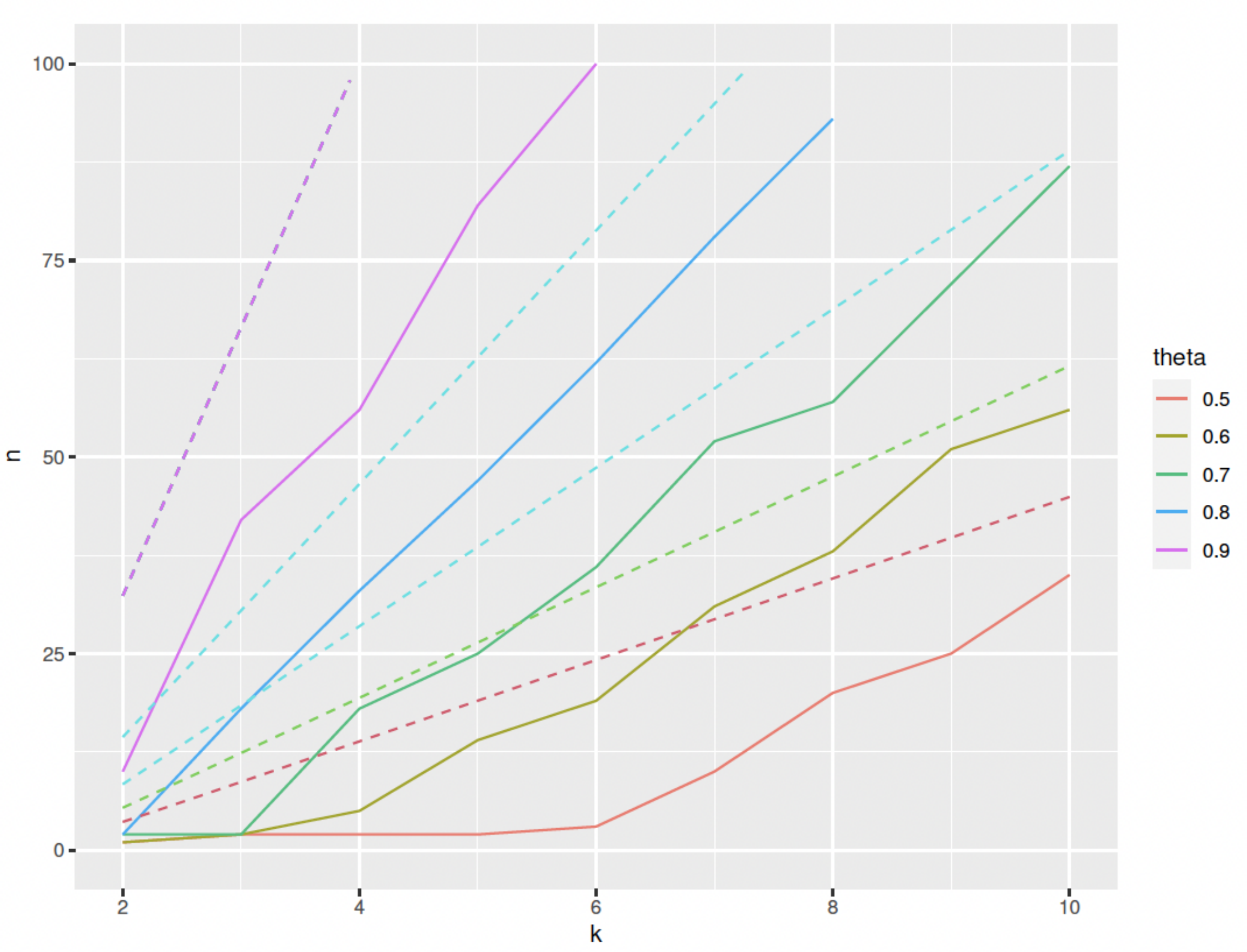


Numerical Simulation

- Choosing option i out of k options yields a reward $x > 0$ with probability p_i and a reward $y = 0$ with probability $1 - p_i$.
- The decision-maker selects the choice with the highest probability estimated with the samples.
- Repeating the decision process N times, the required number of samples to satisfy $P(S) \geq \theta$ is calculated as follows.
  - $n^* = min\left\{n|P(\bar{S}) = N_s(k,n)/N > \theta\right\}$
    - $N_s(n,k)$: the number of trials with the best choice
    - $P(\bar{S}) = N_s(k,n)/N$

Result

- The analytical result of the number of samples (dashed line) is larger than the actual number of samples required (solid line).
- Although there is a difference in the values, there is a similar trend.



Summary and Future Work

- Summary
  - We model small sample-based decision-making by quantum decision-making for network control with a small sample
  - The state is updated by a process of quantum amplification of probability amplitudes for the choices associated with the sampled values
- Future Work
  - We will work on the application of the proposed model for decision making with a small number of samples in actual network control